Review of a few results at the interplay between QG and QI

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TOPICS

- **Introduction**
- **Gauge protected entanglement**
- **Geodesic deviation**
- **Further topics**

INTRODUCTION

Motivation for studying the interplay between QG and QI:

- important foundational notions in QI: entanglement, decoherence, measurement;
- important foundational notions in GR: diffeomorphism invariance, background independence, equivalence principle;
- combining these principles in a full QG theory may lead to unexpected consequences;
- a lot of interest in QG by the QI community…

We review two results:

- N. Pauneković and M. Vojinović,

  "Gauge protected entanglement between gravity and matter",

  Class. Quant. Grav. **35**, 185015 (2018), `[arXiv:1702.07744]`.

- F. Pipa, N. Paunković and M. Vojinović,

  "Entanglement-induced deviation from the geodesic motion in quantum gravity"

  Jour. Cosmol. Astropart. Phys. **09**, 057 (2019), `[arXiv:1801.03207]`.

ENTANGLEMENT

Statement in brief:

- couple matter to gravity (minimal coupling, equivalence principle),
- perform the quantization of the gravity-matter system (or imagine someone else did),
- look for physical, gauge-invariant states (diff-invariance, principle of general relativity),
- for all such states, **gravity and matter are entangled!**

Symbolically, for $\mathcal{H} = \mathcal{H}_G \otimes \mathcal{H}_M$,

$$|\Psi_{\text{physical}}\rangle = c_1|g_1\rangle \otimes |\phi_1\rangle + c_2|g_2\rangle \otimes |\phi_2\rangle + \dots$$

Formally, the gauge-invariant subspace $\mathcal{H}_{\text{phys}}$ of the total Hilbert space $\mathcal{H}$ contains no separable states:

$$(\forall|\Psi\rangle \in \mathcal{H}_{\text{phys}}) \qquad |\Psi\rangle \neq |g\rangle \otimes |\phi\rangle$$

(except maybe by accident).

ENTANGLEMENT

Setup and the classical theory:

- Start from some action for gravity and matter,

$$S[g,\phi] = S_G[g] + S_M[g,\phi]\,,$$

- introduce momenta for fundamental variables g and ϕ,

$$\pi_g \equiv \frac{\delta S}{\delta \partial_0 g}\,, \qquad \pi_\phi \equiv \frac{\delta S}{\delta \partial_0 \phi}\,,$$

- perform the Dirac analysis for constrained systems to find the Hamiltonian in the form

$$H = \int_{\Sigma_3} d^3\vec{x}\, \left[N\mathcal{C} + N^i \mathcal{C}_i + N^{ab}\mathcal{C}_{ab} \right]\,,$$

- where the constraints are the ten generators of the local Poincaré symmetry, in the form:

$$\begin{array}{rcl}
\mathcal{C} & = & \mathcal{C}^G(g,\pi_g) + \mathcal{C}^M(g,\pi_g,\phi,\pi_\phi)\,, \\
\mathcal{C}_i & = & \mathcal{C}_i^G(g,\pi_g) + \mathcal{C}_i^M(g,\pi_g,\phi,\pi_\phi)\,, \\
\mathcal{C}_{ab} & = & \mathcal{C}_{ab}^G(g,\pi_g) + \mathcal{C}_{ab}^M(g,\pi_g,\phi,\pi_\phi)\,.
\end{array}$$

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Canonical quantization (Heisenberg picture):

- promote gravitational and matter fields to operators,

$$
\begin{aligned}
g &\to \hat{g}\,, \qquad & \pi_g &\to \hat{\pi}_g\,, \\
\phi &\to \hat{\phi}\,, \qquad & \pi_\phi &\to \hat{\pi}_\phi\,,
\end{aligned}
$$

- promote Dirac brackets to commutators,

$$
\{\,\cdot\,,\,\cdot\,\}_D \to [\,\cdot\,,\,\cdot\,]\,,
$$

- impose Gupta-Bleuler-like conditions for the state vectors:

$$
\hat{\mathcal{C}}|\Psi\rangle = 0\,, \qquad \hat{\mathcal{C}}_i|\Psi\rangle = 0\,, \qquad \hat{\mathcal{C}}_{ab}|\Psi\rangle = 0\,.
$$

Make sure everything is well defined, unique, etc...

ENTANGLEMENT

Study the structure of the constraint equations:

- the matter-parts of the 3-diffeo and local Lorentz constraints have the benign form

$$\mathcal{C}_i^M = \pi_\phi \nabla_i \phi \,, \qquad \mathcal{C}_{ab}^M = \pi_\phi M_{ab} \phi \,,$$

- while the matter-part of the scalar constraint features the matter Lagrangian:

$$\mathcal{C}^M = \pi_\phi \nabla_\perp \phi - \frac{1}{N} \mathcal{L}_M(g, \pi_g, \phi, \pi_\phi) \,.$$

- Choose some nice representation,

$$\langle g | \hat{g} = g \langle g | \,, \qquad \langle g | \hat{\pi}_g = -i \frac{\delta}{\delta g} \langle g | \,, \qquad \langle \phi | \hat{\phi} = \phi \langle \phi | \,, \qquad \langle \phi | \hat{\pi}_\phi = -i \frac{\delta}{\delta \phi} \langle \phi | \,,$$

- and rewrite the scalar constraint as a functional differential equation:

$$\left[\mathcal{C}^G(g, \tfrac{\partial}{\partial g}) + \mathcal{C}^M(g, \tfrac{\partial}{\partial g}, \phi, \tfrac{\partial}{\partial \phi}) \right] \Psi[g, \phi] = 0 \,.$$

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Look for separable solutions:

$$|\Psi\rangle = |\Psi_G\rangle \otimes |\Psi_M\rangle \qquad \Rightarrow \qquad \Psi[g,\phi] = \Psi_G[g]\Psi_M[\phi]\,.$$

But the scalar constraint equation does not have any such solutions!!!

Namely, if the scalar constraint equation allows for the separation of variables, the matter-part must have the form

$$\mathcal{C}^M(g, \tfrac{\partial}{\partial g}, \phi, \tfrac{\partial}{\partial \phi}) = \mathcal{K}_G(g, \tfrac{\partial}{\partial g})\, \mathcal{K}_M(\phi, \tfrac{\partial}{\partial \phi})\,,$$

but the inspection of Lagrangians shows that it <u>does not</u> have the required form:

$$\mathcal{L}_M^{\text{scalar}} = \sqrt{-g}\left[g^{\mu\nu}(\partial_\mu\varphi)(\partial_\nu\varphi) - m^2\varphi^2 + U(\varphi) \right],$$

$$\mathcal{L}_M^{\text{Dirac}} = \sqrt{-g}\left[\frac{i}{2}\bar{\psi}\gamma^a e^\mu{}_a \nabla_\mu \psi - m\bar{\psi}\psi + \text{c.c.} \right],$$

$$\mathcal{L}_M^{\text{Yang-Mills}} = \sqrt{-g}\left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma}\, \text{tr}\, F_{\mu\nu}F_{\rho\sigma} \right].$$

ENTANGLEMENT

Conclusion:

Separable states are not gauge invariant!!!

What about the other two constraints?

- the local Lorentz constraint does admit separable states as solutions,

$$\mathcal{C}^M_{ab} = \mathcal{C}^M_{ab}(\phi, \pi_\phi) = \pi_\phi M_{ab} \phi \,,$$

- the 3-diffeo constraint admits separable states for the scalar field,

$$\mathcal{C}^M_i(\varphi, \pi_\varphi) = \pi_\varphi \partial_i \varphi \,,$$

  but not for fields of higher spin,

$$\mathcal{C}^M_i(\psi, \pi_\psi, \underbrace{e^i{}_\mu, \omega^{ab}{}_\mu}_{g,\pi_g}) = \pi_\psi \nabla_i \psi \,.$$

- Similarly for internal gauge symmetries such as $SU(3) \times SU(2) \times U(1) \ldots$

ENTANGLEMENT

Main result:

Matter and gravity are always entangled!

Consequences:

- Matter is never in a pure state — after tracing out the gravitational degrees of freedom, the reduced density matrix for matter fields is never a pure state.
- Important for the study of decoherence — one usually starts from an initial separable state, which becomes entangled over time. But even the initial state cannot be separable.
- Gauge-protected entanglement leads to effective "exchange interaction" due to the overlap between states, like for the Pauli exclusion principle. This effective interaction gives rise to deviations from geodesic trajectories for particles, introducing small violation of the weak equivalence principle.
- Numerical calculations in toy-models suggest that the amount of entanglement is rather small, compatible with the semiclassical picture of spacetime.

GEODESICS

Construct a point particle in field theory:

- assume the stress-energy tensor is localized along a timelike trajectory $x^\mu = z^\mu(\tau)$,
- expand it into δ-series around that trajectory,

$$T^{\mu\nu}(x) = \int_{\mathcal{C}} d\tau \left[B^{\mu\nu}(\tau) \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}} + \nabla_\rho \left(B^{\mu\nu\rho}(\tau) \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}} \right) + \dots \right] ,$$

- and approximate the series at the *single-pole* level:

$$T^{\mu\nu}(x) = \int_{\mathcal{C}} d\tau \, B^{\mu\nu}(\tau) \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}} .$$

Then, if matter fields obey local Poincare symmetry, the covariant conservation of stress-energy $\nabla_\nu T^{\mu\nu} = 0$ implies:

- tangent vector $u^\mu \equiv \frac{dz^\mu(\tau)}{d\tau}$ of the trajectory satisfies the geodesic equation: $u^\lambda \nabla_\lambda u^\mu = 0$,
- structure of the stress-energy tensor has the form: $B^{\mu\nu}(\tau) = m \, u^\mu(\tau) u^\nu(\tau)$.

GEODESICS

As we have seen previously, matter and gravity have to be entangled! Therefore, start from a toy-example state:

$$|\boldsymbol{\Psi}\rangle = \alpha \underbrace{|g\rangle \otimes |\phi\rangle}_{|\Psi\rangle} + \beta \underbrace{|\tilde{g}\rangle \otimes |\tilde{\phi}\rangle}_{|\tilde{\Psi}\rangle}, \qquad \alpha \sim 1, \quad \beta \ll 1.$$

Introduce metric and connection operators, $\hat{g}_{\mu\nu} = \hat{g}_{\mu\nu}(\hat{g}, \hat{\pi}_g)$**,** $\hat{\Gamma}^{\lambda}{}_{\mu\nu} = \hat{\Gamma}^{\lambda}{}_{\mu\nu}(\hat{g}, \hat{\pi}_g)$**, and choose states so that the expectation values satisfy classical EoMs in separate branches:**

$$g_{\mu\nu} = \langle \Psi | \hat{g}_{\mu\nu} | \Psi \rangle, \qquad T_{\mu\nu} = \langle \Psi | \hat{T}_{\mu\nu} | \Psi \rangle, \qquad R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g) = 8\pi l_p^2 \, T_{\mu\nu},$$

$$\tilde{g}_{\mu\nu} = \langle \tilde{\Psi} | \hat{g}_{\mu\nu} | \tilde{\Psi} \rangle, \qquad \tilde{T}_{\mu\nu} = \langle \tilde{\Psi} | \hat{T}_{\mu\nu} | \tilde{\Psi} \rangle, \qquad R_{\mu\nu}(\tilde{g}) - \frac{1}{2} \tilde{g}_{\mu\nu} R(\tilde{g}) = 8\pi l_p^2 \, \tilde{T}_{\mu\nu}.$$

Define $|\Psi^{\perp}\rangle$ **to rewrite the state into the form** $|\boldsymbol{\Psi}\rangle = \kappa |\Psi\rangle + \eta |\Psi^{\perp}\rangle$**, where** $\kappa \equiv \alpha + \beta S$**,** $\eta \equiv \beta \sqrt{1 - |S|^2}$**,** $S \equiv \langle \Psi | \tilde{\Psi} \rangle$**. Then expand for** $\eta \to 0$**:**

$$\boldsymbol{g}_{\mu\nu} = \langle \boldsymbol{\Psi} | \hat{g}_{\mu\nu} | \boldsymbol{\Psi} \rangle = g_{\mu\nu} + \eta \, h_{\mu\nu} + \mathcal{O}(\eta^2), \qquad h_{\mu\nu} \equiv 2 \, \mathrm{Re} \left(\kappa \langle \Psi^{\perp} | \hat{g}_{\mu\nu} | \Psi \rangle \right),$$

$$\boldsymbol{T}_{\mu\nu} = \langle \boldsymbol{\Psi} | \hat{T}_{\mu\nu} | \boldsymbol{\Psi} \rangle = T_{\mu\nu} + \eta \, t_{\mu\nu} + \mathcal{O}(\eta^2), \qquad t_{\mu\nu} \equiv 2 \, \mathrm{Re} \left(\kappa \langle \Psi^{\perp} | \hat{T}_{\mu\nu} | \Psi \rangle \right).$$

GEODESICS

If local Poincaré invariance is preserved at the quantum level, it gives rise to a Gupta-Bleuler-like condition:

$$\langle \mathbf{\Psi} | \hat{\nabla}_\nu \hat{T}^{\mu\nu} | \mathbf{\Psi} \rangle = 0 \, ,$$

Use a sequence of approximations (see the paper for details) to reduce the operator equation to the corresponding effective classical equation,

$$\boldsymbol{\nabla}_\nu \boldsymbol{T}^{\mu\nu} = 0 \, ,$$

where both stress-energy and the covariant derivative are expressed as expectation values:

$$\boldsymbol{T}^{\mu\nu} \equiv \langle \mathbf{\Psi} | \hat{T}^{\mu\nu} | \mathbf{\Psi} \rangle \, , \qquad \boldsymbol{\nabla}_\nu \equiv \langle \mathbf{\Psi} | \hat{\nabla}_\nu | \mathbf{\Psi} \rangle \, .$$

Like stress-energy, expand the Christoffel symbol into η-series, since it is a function of the metric:

$$\boldsymbol{\Gamma}^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} + \eta F^\lambda{}_{\mu\nu} + \mathcal{O}(\eta^2) \, , \qquad F^\mu{}_{\nu\sigma} \equiv \nabla_{(\sigma} h^\mu{}_{\nu)} - \frac{1}{2} \nabla^\mu h_{\nu\sigma} \, ,$$

The result is the effective covariant conservation equation — with an η-correction term:

$$\nabla_\nu \left(T^{\mu\nu} + \eta t^{\mu\nu} \right) + 2\eta F^{(\mu}{}_{\nu\sigma} T^{\nu)\sigma} = 0 \, .$$

GEODESICS

Now employ the effective covariant conservation to obtain the equation of motion for the particle, like in the classical case — expand them into δ-series and approximate at the single-pole level:

$$T^{\mu\nu}(x) = \int_{\mathcal{C}} d\tau B^{\mu\nu}(\tau) \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}}\,, \qquad t^{\mu\nu}(x) = \int_{\mathcal{C}} d\tau \bar{B}^{\mu\nu}(\tau) \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}}\,.$$

The results:

- the modified geodesic equation:

$$\nabla u^{\mu} + \eta u^{\nu} u^{\sigma} F^{\mu}_{\perp\nu\sigma} = 0\,, \qquad F^{\mu}_{\perp\nu\sigma} \equiv P^{\mu}_{\perp\lambda} F^{\lambda}{}_{\nu\sigma}\,,$$

- equation for the structure of effective stress-energy tensor:

$$B^{\mu\nu} + \eta \bar{B}^{\mu\nu} = m(\tau) u^{\mu} u^{\nu}\,,$$

- nonrelativistic (Newtonian) limit of the equation of motion:

$$m_I \frac{d^2 z_k}{d\tau^2} = -\underbrace{m_I \left(1 - \frac{1}{3}\eta h^i{}_i\right)}_{m_G} \frac{GM}{r^3} z_k - \eta m_I \left[\partial_0 h_{0k} - \frac{1}{2}\partial_k h_{00} - \frac{GM}{r^3} z^j \tilde{h}_{jk}\right]\,,$$

where $h^i{}_i = 2\delta^{ij}\,\mathrm{Re}\left(\kappa\langle\Psi^{\perp}|\hat{g}_{ij}|\Psi\rangle\right) + \mathcal{O}(\eta)$.

FURTHER TOPICS

- geodesic deviation equation
- analysis of the weak and strong equivalence principles
- equation of motion at η^2 order
- more "serious" superpositions of gravity — multiple causal orders, closed timelike curves, ...
- operational approach to measuring the spacetime manifold
- etc...

THANK YOU!