

# Operational existence of a spacetime manifold

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# OVERVIEW

- Introduction
- Mechanics and the time manifold
- Field theory and the spacetime manifold
- Conclusions and further topics

# INTRODUCTION

Begin by looking at a metaphysics list of Frequently Asked Questions:

- What is time? What is space?
- Do time and space really exist?
- Does spacetime have an ontological meaning?
- Is spacetime imagined, invented, postulated, discovered, ...?
- Who wins — substantivalists or relationalists? :-)

I'll attempt to answer at least some of these questions, in three steps:

- start from a relational point of view — a priori no objective space or time,
- introduce a simple mechanical gedanken-experiment  $\Rightarrow$  **time manifold!**
- enlarge the gedanken-experiment to field theory level  $\Rightarrow$  **spacetime manifold!**

For more details, see [arXiv:2209.04783](https://arxiv.org/abs/2209.04783)

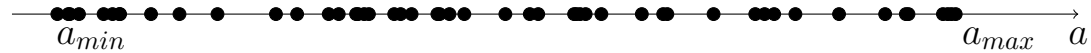
# MECHANICS and TIME

Gedanken-experiment:

- describe a motion of a pendulum (called  $A$ )
- randomly take  $N$  still photos of the pendulum
- for every photo, measure the distance from the vertical axis
- make sure NOT to remember the order of measured results (no cheating about time!!)

$$A = \{a_1, \dots, a_N\} \subset S_A = [a_{min}, a_{max}]$$

- draw the results on a scatter plot:



Outcome — not too informative...

# MECHANICS and TIME

**Extend the gedanken-experiment:**

- let's go relational — describe a motion of a pendulum  $A$  *with respect to* the pendulum  $B$
- randomly take  $N$  still photos of both pendulums side-by-side
- cut the photos, so that each half displays only  $A$  or  $B$  independently
- measure the distance from the vertical axis for both  $A$  and  $B$
- make sure NOT to remember the order of measured results, *nor the pairings* (no cheating!!)

$$\begin{aligned} A &= \{a_1, \dots, a_N\} \subset S_A = [a_{min}, a_{max}] \\ B &= \{b_1, \dots, b_N\} \subset S_B = [b_{min}, b_{max}] \end{aligned}$$

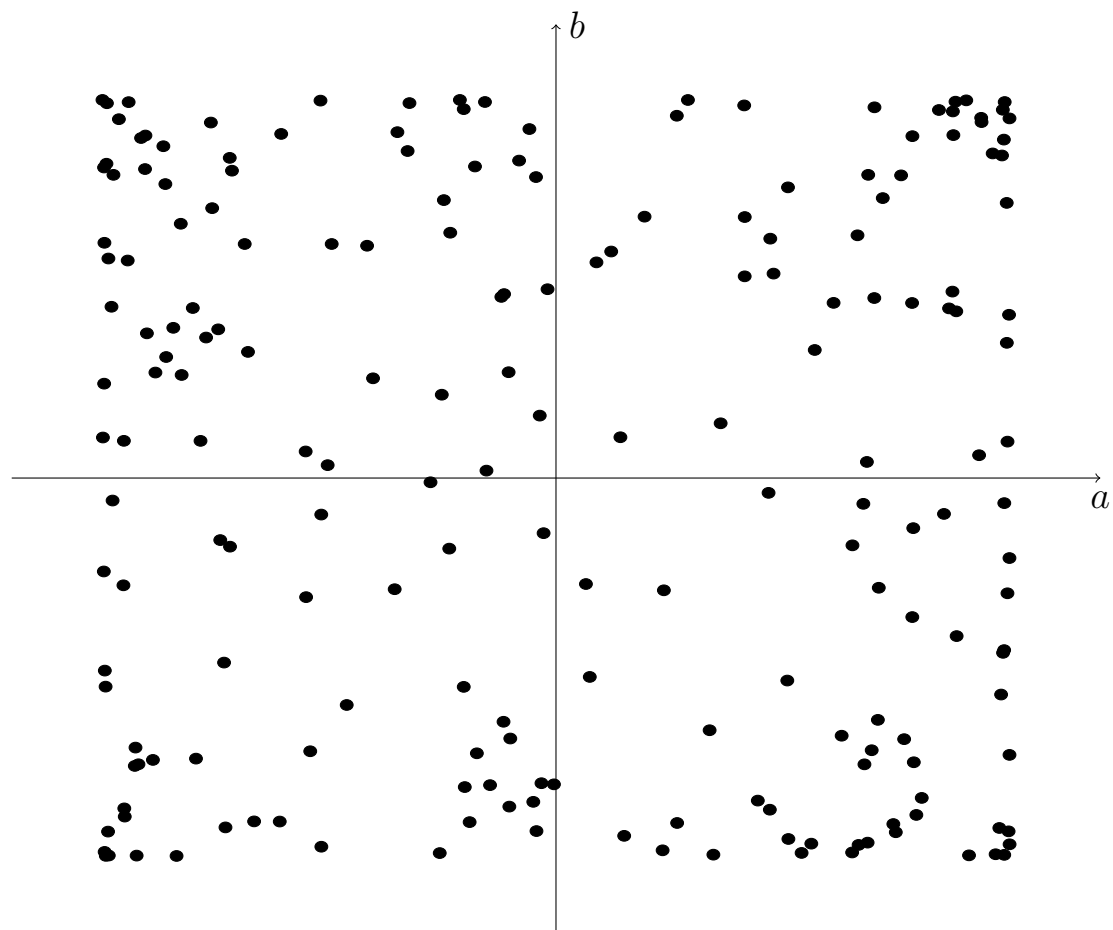
- randomly pair up each  $a_i$  with some  $b_{\pi(i)}$  into a permutation matrix

$$\begin{pmatrix} a_1 & a_2 & \dots & a_N \\ b_{\pi(1)} & b_{\pi(2)} & \dots & b_{\pi(N)} \end{pmatrix},$$

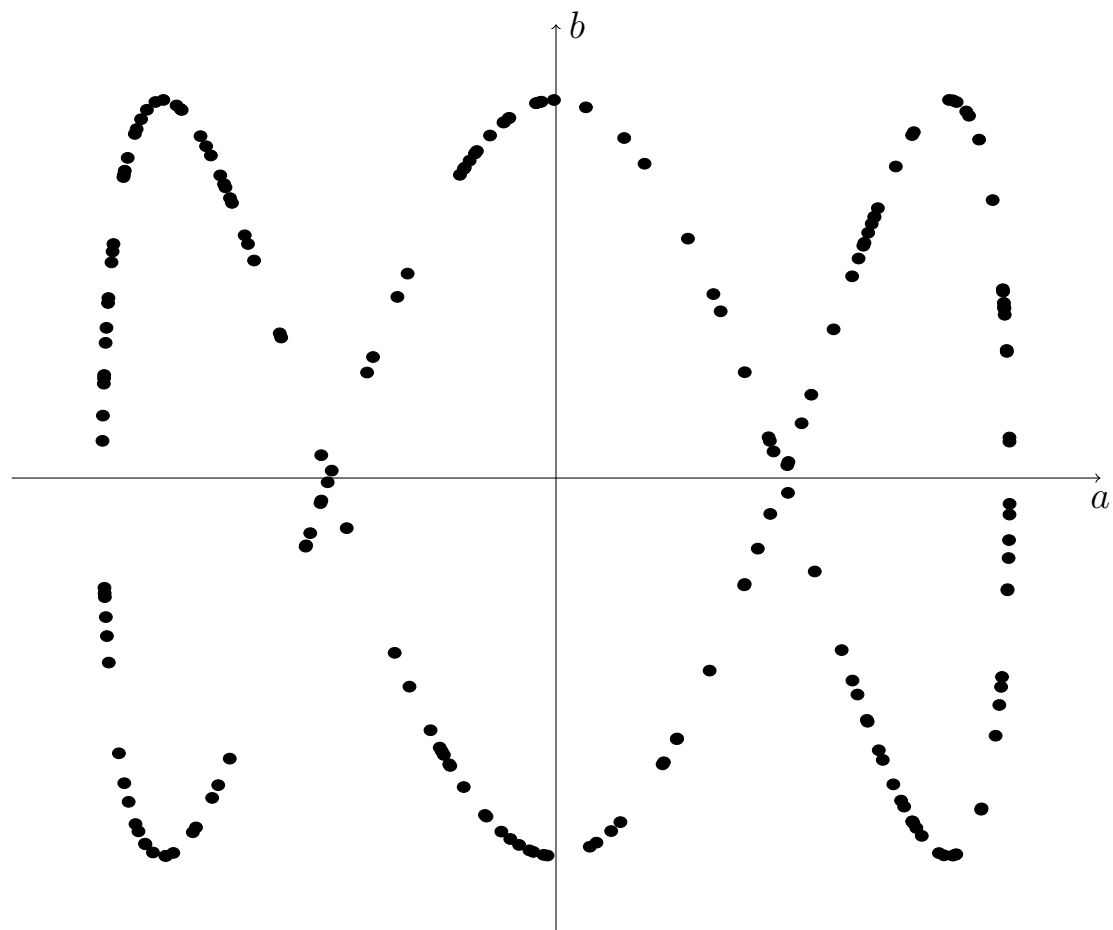
to obtain the coordinates  $(a_i, b_{\pi(i)})$  for the 2D graph  $S_A \times S_B$

- plot them!

# MECHANICS and TIME



# MECHANICS and TIME



# MECHANICS and TIME

Was this an accident? Extend the gedanken-experiment yet again:

- describe a motion of *three* pendulums,  $A$ ,  $B$  and  $C$
- randomly take  $N$  still photos of all three pendulums side-by-side
- cut the photos, so that each piece displays only  $A$  or  $B$  or  $C$  independently
- measure the distances, forgetting all orders of measured results

$$\begin{aligned} A &= \{a_1, \dots, a_N\} \subset S_A = [a_{min}, a_{max}] \\ B &= \{b_1, \dots, b_N\} \subset S_B = [b_{min}, b_{max}] \\ C &= \{c_1, \dots, c_N\} \subset S_C = [c_{min}, c_{max}] \end{aligned}$$

- randomly construct triplets of  $a_i$ ,  $b_{\pi(i)}$  and  $c_{\rho(i)}$  into a double-permutation matrix

$$\begin{pmatrix} a_1 & a_2 & \dots & a_N \\ b_{\pi(1)} & b_{\pi(2)} & \dots & b_{\pi(N)} \\ c_{\rho(1)} & c_{\rho(2)} & \dots & c_{\rho(N)} \end{pmatrix},$$

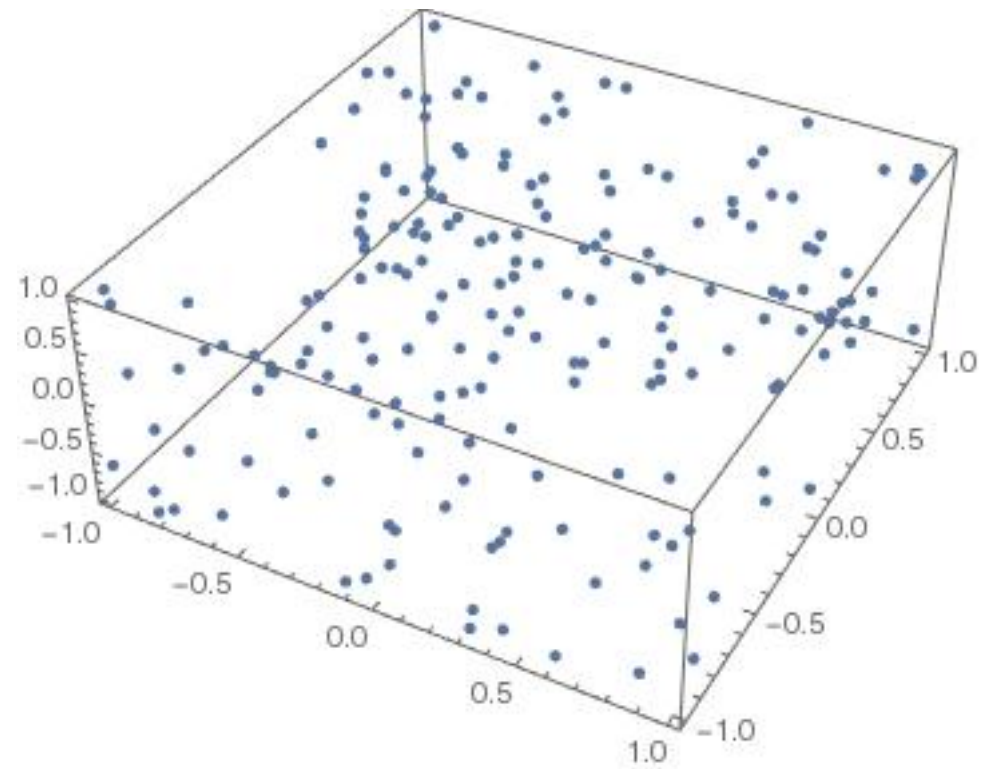
to obtain the coordinates  $(a_i, b_{\pi(i)}, c_{\rho(i)})$  for the 3D graph  $S_A \times S_B \times S_C$

- and now plot that!



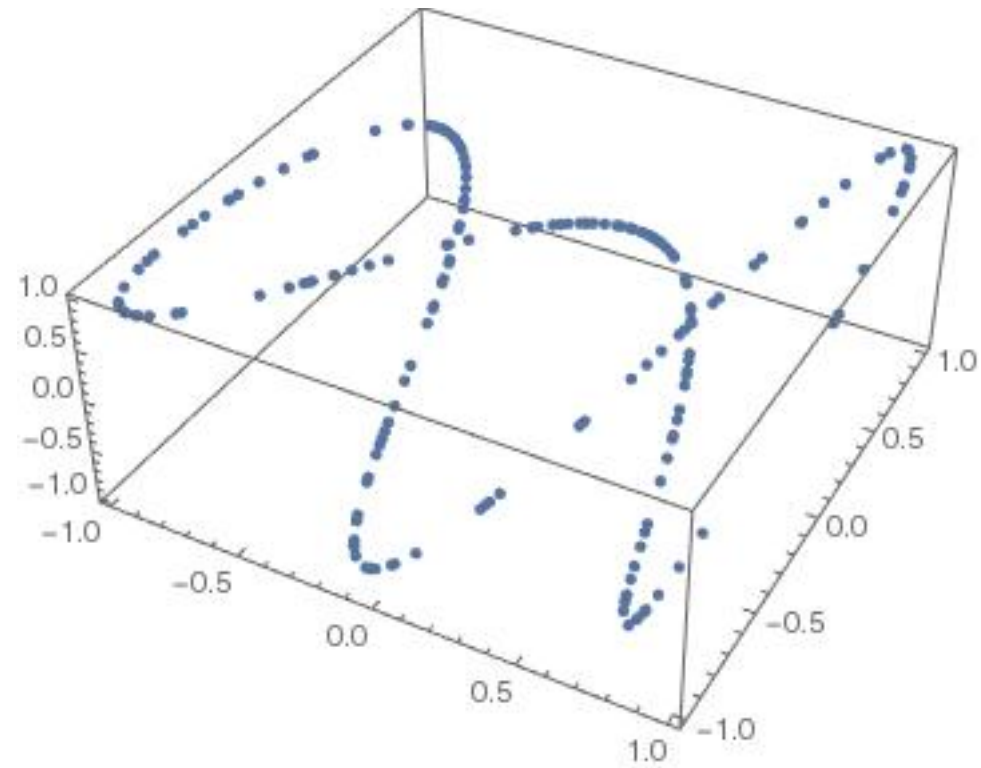
# MECHANICS and TIME

A scatter plot of an ordinary permutation:



# MECHANICS and TIME

A scatter plot of an extraordinary permutation:



# MECHANICS and TIME

## Four main properties of the correlation graph:

- *existence* — any set of real-world mechanical observables gives rise to the signal-permutation graph, called  $\mathcal{M}$ , such that the points lie on a manifold of smaller dimension, i.e.,

$$\mathcal{M} \subset S_A \times S_B \times \dots, \quad \frac{\text{meas}(\mathcal{M})}{\text{meas}(S_A \times S_B \times \dots)} \rightarrow 0 \quad (N \rightarrow \infty).$$

- *self-reinforcement* — adding more datapoints disturbs all ordinary graphs, but reinforces the signal-permutation graph:

$$\Pi(k) = \begin{cases} \pi_1(k), & k \in \{1, \dots, N\} \\ \pi_2(k), & k \in \{N + 1, \dots, N + M\} \end{cases} .$$

Signal-graphs of all subsets of data and their union lie on the same  $\mathcal{M}$ .

- *dimensionality* — adding more observables does not change the dimension of the signal-graph, and the experimental result is that for all mechanical systems we always obtain

$$\dim \mathcal{M} = 1 .$$

- *topology* — slightly more statistical analysis gives rise to a basis of *open sets* on  $\mathcal{M}$ , prescribing its topology (either open or closed curve).

# MECHANICS and TIME

Is  $\mathcal{M}$  really a manifold?

- it is a nonempty set of points,
- it has a well-defined topology, i.e., it is a *topological space*,
- it has a well-defined dimension, i.e., it is a *topological manifold*,
- we can introduce a *coordinate chart*:

$$T : \mathcal{M} \rightarrow \mathbb{R}, \quad (a, b, c, \dots) \mapsto t,$$

and its inverse,

$$T^{-1} : \mathbb{R} \rightarrow \mathcal{M}, \quad t \mapsto (a(t), b(t), c(t), \dots)$$

which represents the *parametric equations of motion* for each of the mechanical observables ,

- it features *diffeomorphism invariance* of a 1-dimensional manifold, i.e., so-called *reparametrization invariance*.

$\Rightarrow$       **TIME MANIFOLD !!!**

# FIELD THEORY and SPACETIME

Extend the gedanken-experiment to include non-mechanical observables:

- consider a fluid in motion, such as a river flowing through a riverbed,
- introduce  $N$  small probes which measure  $K$  different observables:

$$\rho^m, \quad p, \quad T, \quad \rho^e, \quad E, \quad B, \quad \vec{E} \cdot \vec{B}, \quad \dots$$

- let the probes scatter throughout the river, and randomly activate and transmit the measured values to a computer,
- the computer remembers the obtained values of the observables, but *does not remember* their order,

$$\begin{aligned} \rho^m &= \{\rho_1^m, \dots, \rho_N^m\}, \\ p &= \{p_1, \dots, p_N\}, \\ T &= \{T_1, \dots, T_N\}, \\ &\vdots \end{aligned}$$

- construct  $K$ -tuples by pairing the observables using random permutations,
- have the computer memorize those points on a  $K$ -dimensional scatter “plot”.

# FIELD THEORY and SPACETIME

Then discover that:

- there exists a special permutation, which features the four properties of (1) existence, (2) self-reinforcement, (3) dimensionality, and (4) topology,
- call this special permutation  $\mathcal{M}$ , and discover that for all observables, all physical systems, and *all experiments ever done*, it turns out that

$$\dim \mathcal{M} = 4, \quad \mathcal{M} \text{ has simply connected topology}$$

- finally, introduce a chart

$$f : \mathcal{M} \rightarrow \mathbb{R}^4, \quad (\rho^m, p, \dots) \mapsto (t, x, y, z)$$

and its inverse

$$f^{-1} : \mathbb{R}^4 \rightarrow \mathcal{M}, \quad (t, x, y, z) \mapsto \left( \rho^m(t, x, y, z), p(t, x, y, z), \dots \right)$$

- and note that  $\mathcal{M}$  features invariance with respect to  $Diff(\mathbb{R}^4)$ .

$\Rightarrow$       **SPACETIME MANIFOLD !!!**

# CONCLUSION

*There exists a unique signal in experimental data of our gedanken-experiment, that corresponds to the existence of a spacetime manifold, 4-dimensional and simply connected.*

*This signal is of the same quality as the signal for the existence of atoms, electrons, EM-field, and other phenomena in physics (including the Pope in Rome). One can therefore argue that the spacetime manifold is equally objective and ontologically real as these other phenomena.*

Further topics for discussion:

- what about quantum mechanics and non-commuting observables?
- what about extra dimensions of spacetime and experiments (not yet done) at the Planck scale?
- what about global symmetries and uniqueness of the special permutation graph?
- what about the smoothness structure of spacetime?
- what about the notion of “emergence of spacetime”?
- etc. . .

***THANK YOU!***