A note on the equivalence principle in general relativity and Yang-Mills theories

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TOPICS

- **Introduction**
- **EP for gravity**
- **An example**
- **EP for electrodynamics**
- **Another example**
- **Conclusions**

INTRODUCTION

Talk based on the paper:

N. Paunković and MV, *Universe* **8**, 598 (2022). [arXiv:2210.00133]

The paper discusses (among other things) various formulations of EP:

- Equality of gravitational and inertial mass (*Newtonian EP*)
- Universality of free fall (*Galileian EP*)
- Local equality between gravity and inertia (*Einstein's EP*)
- EP applied to mechanical bodies (*Weak EP*)
- EP applied to field theory (*Strong EP*)

All of these versions, except SEP, are known to be violated in nature!

But there is more!! EP can be formulated for Yang-Mills fields as well!!

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A few comments:

- The *purpose* of EP is to prescribe the precise interaction (coupling) between matter and gravity.
- The *local nature* of EP is important, implying that gravitational effects are visible only *nonlocally*, by integrating EoMs for matter.
- EP exactly holds in Nature when applied to *field theory* (SEP).
- When applied to mechanical particles (WEP), EP is known to be violated. Counterexample: Mathisson-Papapetrou EoM for a particle with nonzero angular momentum:

$$u^\mu \nabla_\mu u^\lambda = R^\lambda{}_{\mu\rho\sigma}\, u^\mu J^{\rho\sigma}\,.$$

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$$g_{\mu\nu}(x) \to \tilde{g}_{\mu\nu}(x)\,, \qquad \tilde{g}_{\mu\nu}(x_0) = \eta_{\mu\nu}\,,$$

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where $K \equiv R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$.

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The above statement of EP can be generalized so that it holds not just for gravity, but also for electrodynamics:

The equations of motion for matter coupled to the electromagnetic field remain locally identical to the equations of motion for matter in the absence of the electromagnetic field.

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A few comments:

- In electromagnetic EP, "matter" is assumed to be everything except gravity and EM fields.
- The electromagnetic EP exactly holds in Nature when applied to *field theory* (SEP).
- When applied to mechanical particles (WEP), electromagnetic EP is known to be violated. Counterexample: Lorentz force EoM for a charged particle:

$$u^{\mu}\nabla_{\mu}u^{\lambda} = \frac{q}{m}F^{\lambda}{}_{\mu}\,u^{\mu}\,.$$

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$$\left[i\gamma^\mu\mathcal{D}_\mu - m + I_1 + I_2\right]\psi(x) = 0\,.$$

where $I_1 \equiv F^{\mu\nu}F_{\mu\nu}$ and $I_2 \equiv \varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$.

CONCLUSIONS

- EP does not apply only to the gravitational field, but to EM field as well.
- One can easily verify that EP applies also in the case of non-Abelian gauge fields (Yang-Mills):
$$\lambda^a(x) = -A^a_\mu(x_0)\,(x^\mu - x^\mu_0)\ .$$
- As a consequence, *all four interactions in Nature* (gravitational, EM, weak, strong) obey the EP.
- Equivalent terminology for the statement of EP is the "minimal coupling prescription" (more common in QFT contexts).
- When discussing EP for YM fields, one must keep track of what fields are called "matter".
- SEP holds exactly in Nature, all other variants of EP are known to be violated.

THANK YOU!