Operational verification of the existence of a spacetime manifold

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OVERVIEW

- **Introduction**
- **Mechanics and the time manifold**
- **Field theory and the spacetime manifold**
- **Conclusions and further topics**

INTRODUCTION

Begin by looking at a metaphysics list of Frequently Asked Questions:

- What is time? What is space?
- Do time and space really exist?
- Does spacetime have an ontological meaning?
- Is spacetime imagined, invented, postulated, discovered, . . . ?
- Who wins — substantivalists or relationalists? :-)

I'll attempt to answer at least some of these questions, in three steps:

- start from a relational point of view — a priori no objective space or time,
- introduce a simple mechanical gedanken-experiment ⇒ **time manifold!**
- enlarge the gedanken-experiment to field theory level ⇒ **spacetime manifold!**

For more details, see `arXiv:2209.04783`

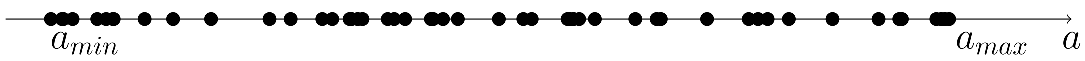
MECHANICS and TIME

Gedanken-experiment:

- describe a motion of a pendulum (called A)
- randomly take N still photos of the pendulum
- for every photo, measure the distance from the vertical axis
- make sure NOT to remember the order of measured results (no cheating about time!!)

$$A = \{a_1, \ldots, a_N\} \subset S_A = [a_{min}, a_{max}]$$

- draw the results on a scatter plot:



Outcome — not too informative. . .

MECHANICS and TIME

Extend the gedanken-experiment:

- let's go relational — describe a motion of a pendulum A *with respect to* the pendulum B
- randomly take N still photos of both pendulums side-by-side
- cut the photos, so that each half displays only A or B independently
- measure the distance from the vertical axis for both A and B
- make sure NOT to remember the order of measured results, *nor the pairings* (no cheating!!)

$$\begin{array}{rclclcl} A &=& \{a_1, \ldots, a_N\} &\subset& S_A &=& [a_{min}, a_{max}] \\ B &=& \{b_1, \ldots, b_N\} &\subset& S_B &=& [b_{min}, b_{max}] \end{array}$$

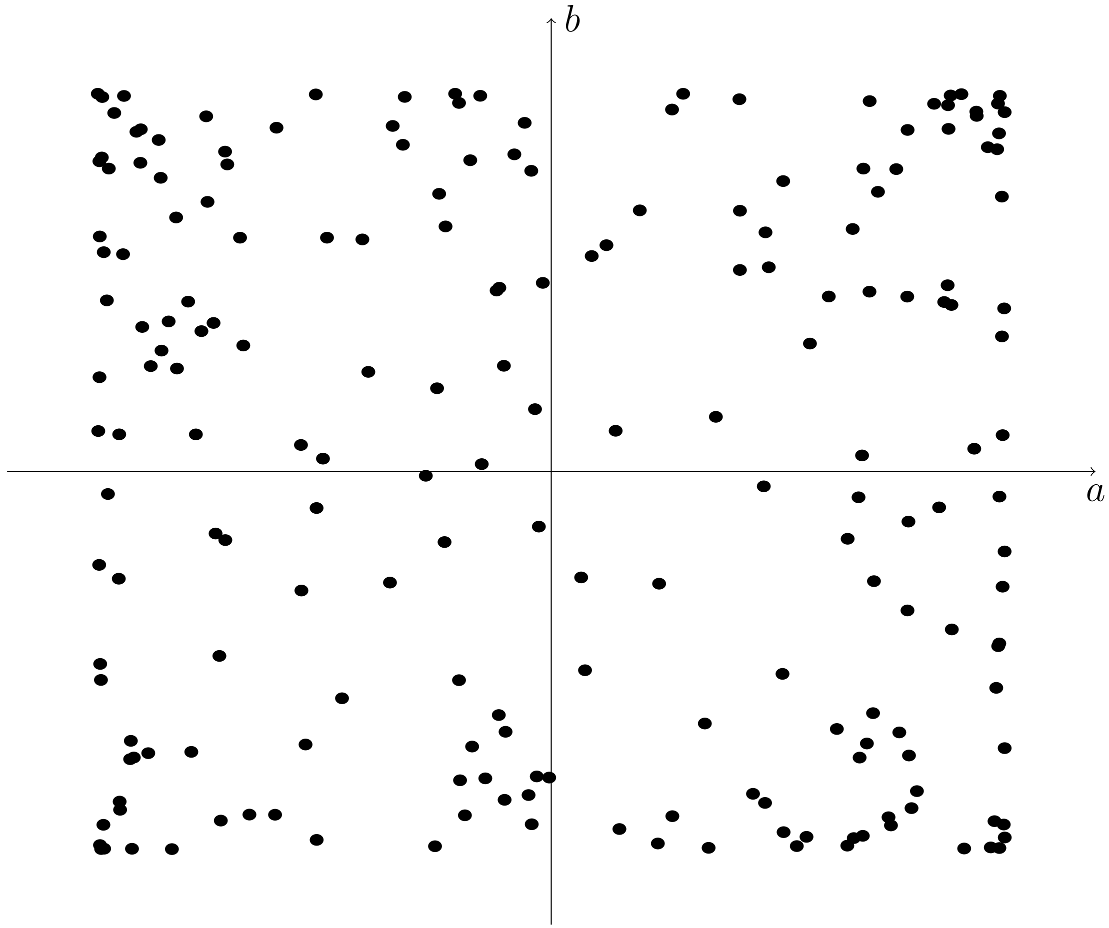
- randomly pair up each a_i with some $b_{\pi(i)}$ into a permutation matrix

$$\begin{pmatrix} a_1 & a_2 & \ldots & a_N \\ b_{\pi(1)} & b_{\pi(2)} & \ldots & b_{\pi(N)} \end{pmatrix},$$

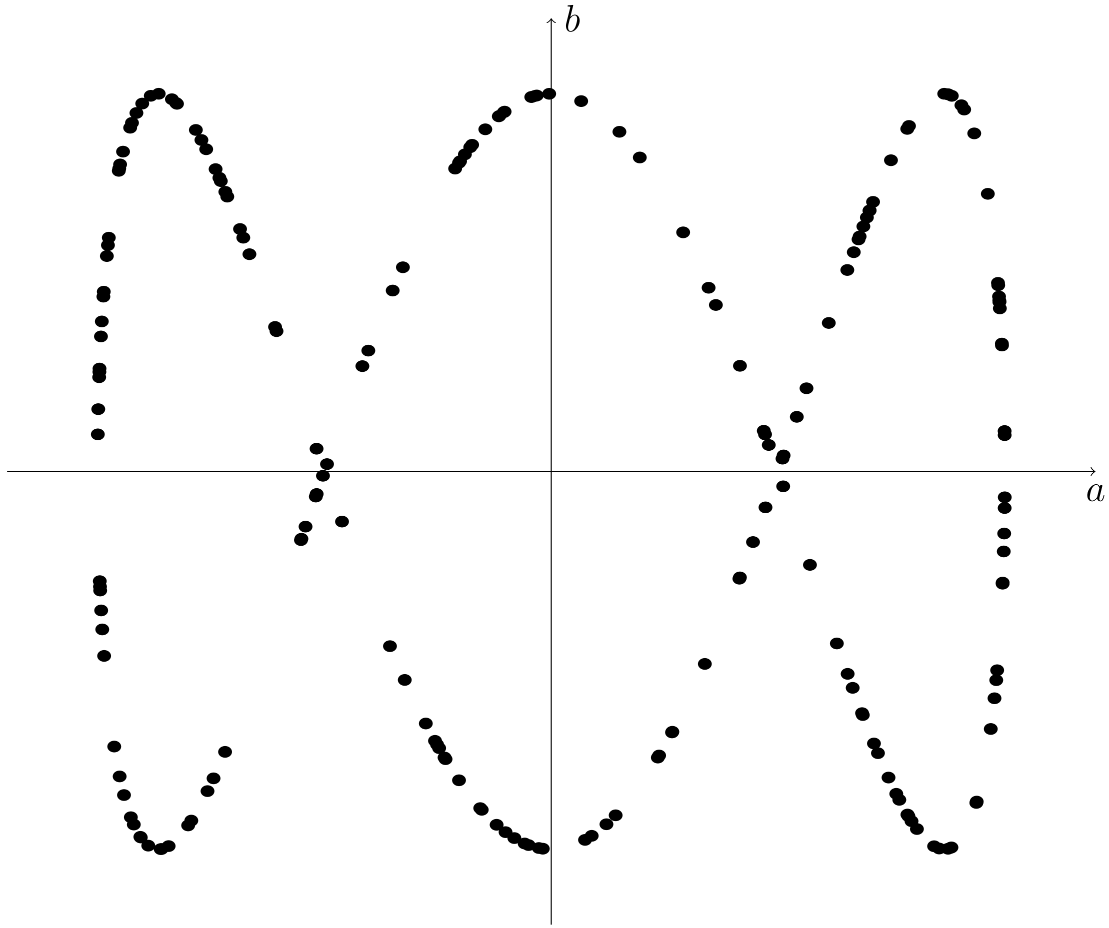
  to obtain the coordinates $(a_i, b_{\pi(i)})$ for the $2D$ graph $S_A \times S_B$

- plot them!

MECHANICS and TIME



MECHANICS and TIME



MECHANICS and TIME

Was this an accident? Extend the gedanken-experiment yet again:

- describe a motion of *three* pendulums, A, B and C
- randomly take N still photos of all three pendulums side-by-side
- cut the photos, so that each piece displays only A or B or C independently
- measure the distances, forgetting all orders of measured results

$$\begin{array}{rcccccl} A & = & \{a_1, \ldots, a_N\} & \subset & S_A & = & [a_{min}, a_{max}] \\ B & = & \{b_1, \ldots, b_N\} & \subset & S_B & = & [b_{min}, b_{max}] \\ C & = & \{c_1, \ldots, c_N\} & \subset & S_C & = & [c_{min}, c_{max}] \end{array}$$

- randomly construct triplets of a_i, $b_{\pi(i)}$ and $c_{\rho(i)}$ into a double-permutation matrix

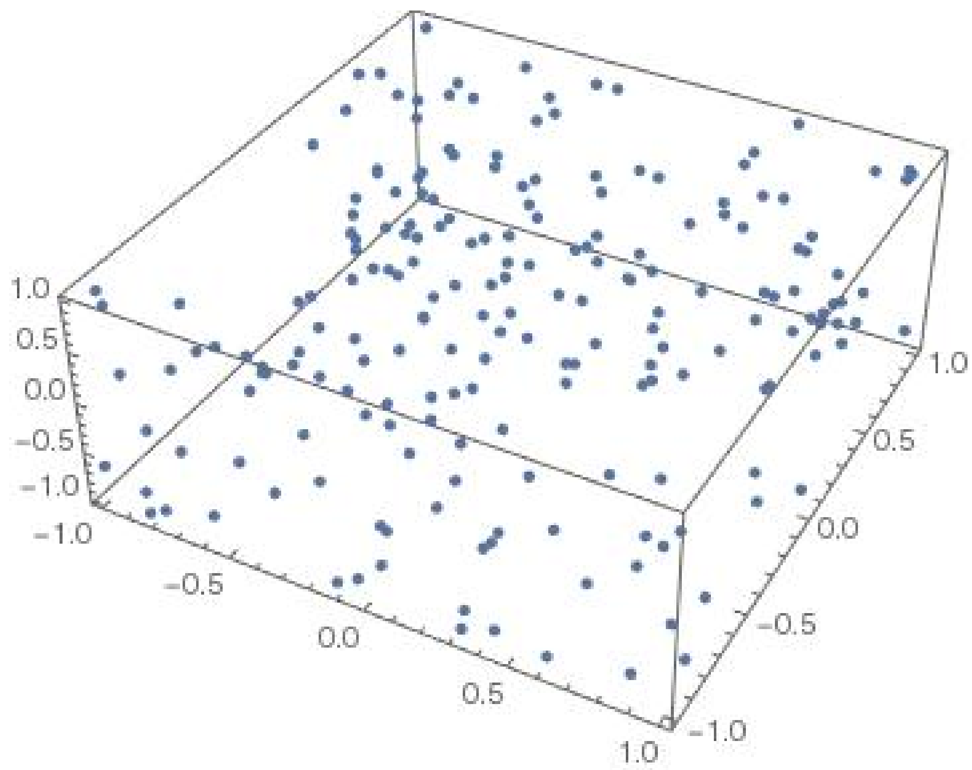
$$\begin{pmatrix} a_1 & a_2 & \ldots & a_N \\ b_{\pi(1)} & b_{\pi(2)} & \ldots & b_{\pi(N)} \\ c_{\rho(1)} & c_{\rho(2)} & \ldots & c_{\rho(N)} \end{pmatrix},$$

  to obtain the coordinates $(a_i, b_{\pi(i)}, c_{\rho(i)})$ for the $3D$ graph $S_A \times S_B \times S_C$

- and now plot that!

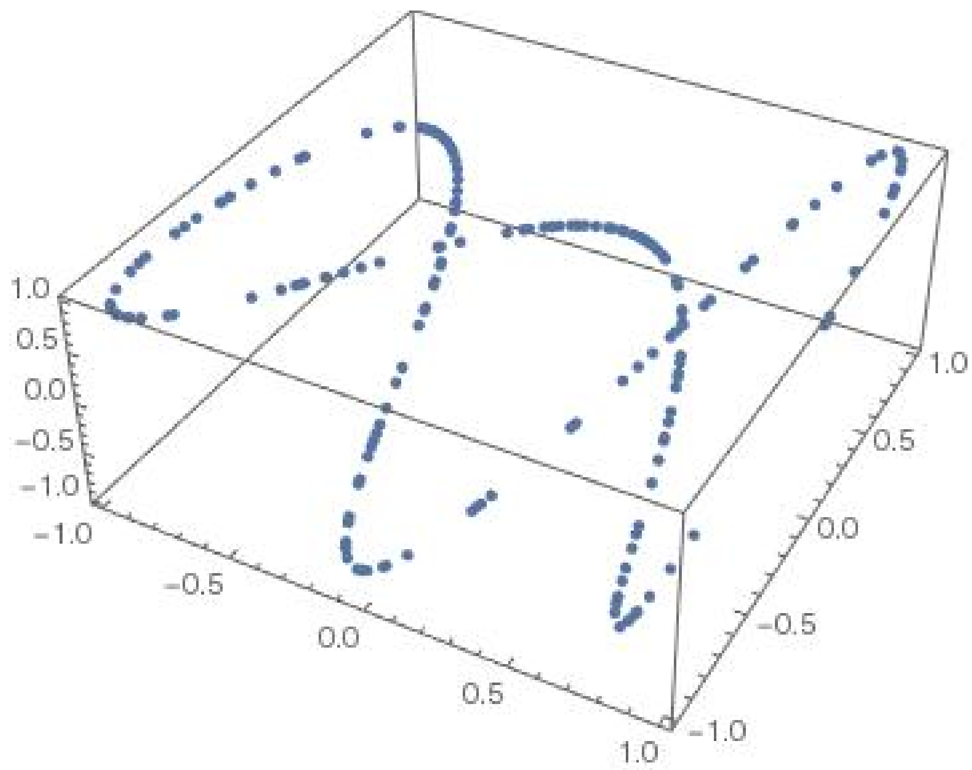
MECHANICS and TIME

A scatter plot of an ordinary permutation:



MECHANICS and TIME

A scatter plot of an extraordinary permutation:



MECHANICS and TIME

Four main properties of the correlation graph:

- *existence* — any set of real-world mechanical observables gives rise to the signal-permutation graph, called $\mathcal{M}$, such that the points lie on a manifold of smaller dimension, i.e.,

$$\mathcal{M} \subset S_A \times S_B \times \dots\,, \qquad \frac{\mathrm{meas}(\mathcal{M})}{\mathrm{meas}(S_A \times S_B \times \dots)} \to 0 \qquad (N \to \infty)\,.$$

- *self-reinforcement* — adding more datapoints disturbs all ordinary graphs, but reinforces the signal-permutation graph:

$$\Pi(k) = \begin{cases} \pi_1(k)\,, & k \in \{1, \dots, N\} \\ \pi_2(k)\,, & k \in \{N+1, \dots, N+M\} \end{cases}\,.$$

  Signal-graphs of all subsets of data and their union lie on the same $\mathcal{M}$.

- *dimensionality* — adding more observables does not change the dimension of the signal-graph, and the experimental result is that for all mechanical systems we always obtain

$$\dim \mathcal{M} = 1\,.$$

- *topology* — slightly more statistical analysis gives rise to a basis of *open sets* on $\mathcal{M}$, prescribing its topology (either open or closed curve).

MECHANICS and TIME

Is $\mathcal{M}$ really a manifold?

- it is a nonempty set of points,
- it has a well-defined topology, i.e., it is a *topological space*,
- it has a well-defined dimension, i.e., it is a *topological manifold*,
- we can introduce a *coordinate chart*:

$$T \;:\; \mathcal{M} \to \mathbb{R}\,, \qquad (a, b, c, \dots) \mapsto t\,,$$

  and its inverse,

$$T^{-1} \;:\; \mathbb{R} \to \mathcal{M}\,, \qquad t \mapsto (\, a(t),\, b(t),\, c(t),\, \dots\,)$$

  which represents the *parametric equations of motion* for each of the mechanical observables ,
- it features *diffeomorphism invariance* of a 1-dimensional manifold, i.e., so-called *reparametrization invariance.*

$\Rightarrow$ ***TIME MANIFOLD !!!***

FEW TECHNICAL DETAILS

An overview of the general algorithm to determine the properties of a submanifold structure in the dataset:

- given a set of N points scattered in a configuration space of volume V_K, construct a K-dimensional cube around each point, of edge length $\epsilon = \sqrt[K]{\frac{V_K}{N}}$,
- the total volume of all cubes is then

$$V_{\text{total}} = \sum_{n=1}^{N} \left(V_{\text{cube}} - V_{\text{overlap}}^{(n)} \right) = V_K - \sum_{n=1}^{N} V_{\text{overlap}}^{(n)} = \alpha(N) V_K \, ,$$

- the so-called *critical parameter* is thus defined as

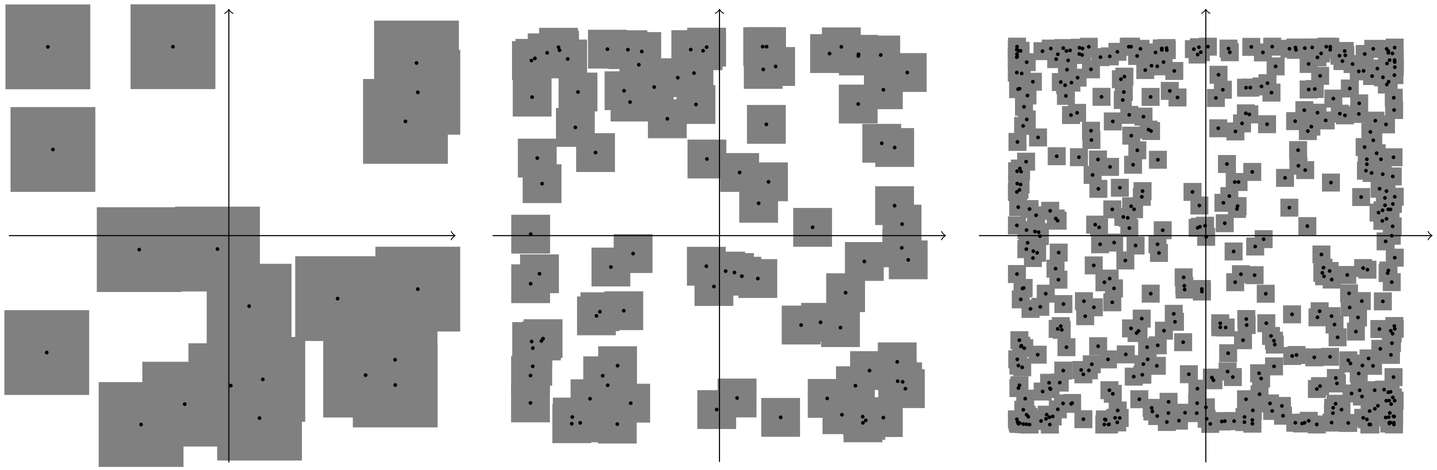
$$\alpha(N) = 1 - \frac{1}{V_K} \sum_{n=1}^{N} V_{\text{overlap}}^{(n)} \, ,$$

  which is always in the range $0 \leqslant \alpha \leqslant 1$.

If the overlap is small, we have $\alpha \sim 1$, while if the overlap is large, we have $\alpha \sim 0$.

FEW TECHNICAL DETAILS

A generic random permutation, for an increasing number of datapoints, featuring small-to-average overlap volume:

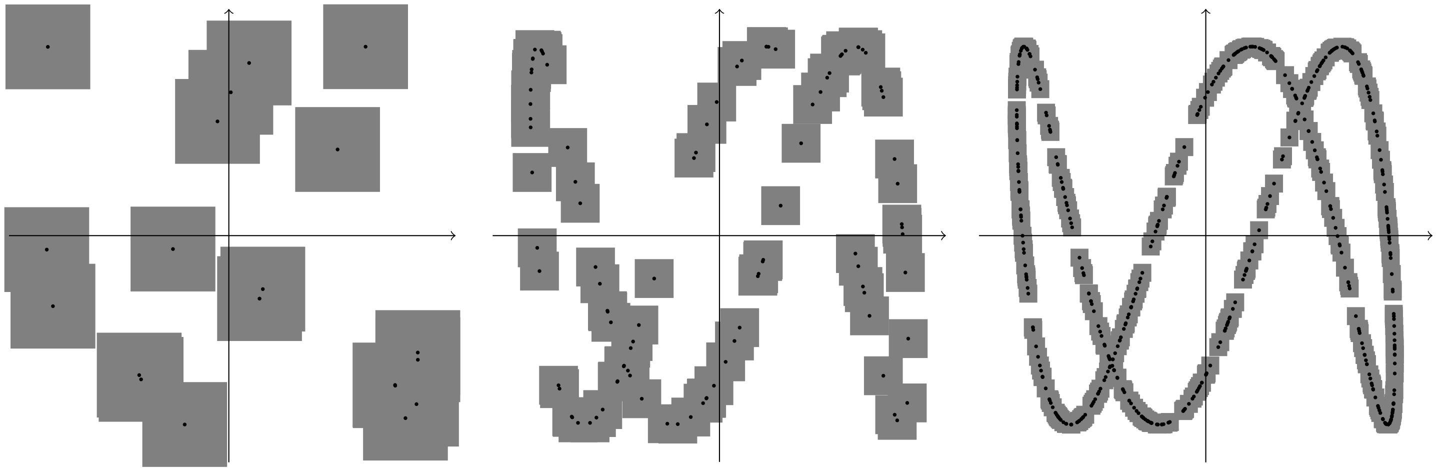


$N = 20$ $N = 100$ $N = 500$

One obtains that $\alpha(N)$ remains finite as N grows.

FEW TECHNICAL DETAILS

The extraordinary permutation, for an increasing number of datapoints, featuring very high overlap volume:



$N = 20$ $N = 100$ $N = 500$

One obtains that $\alpha(N) \sim N^{\frac{D}{K}-1}$ as N grows, dropping to zero for $D < K$, where D is the dimension of the hypersurface $\mathcal{M}$.

FEW TECHNICAL DETAILS

The overlap volume, and thus $\alpha(N)$, can be numerically evaluated for any dataset. If the dataset corresponds to the extraordinary permutation, the points will align themselves on some hypersurface:

- the hypersurface will in general be of zero measure, in the limit $N \gg 1$,

$$\frac{\mathrm{meas}(\mathcal{M})}{\mathrm{meas}(\mathrm{box})} = \lim_{N\to\infty} \frac{V_{\mathrm{total}}}{V_K} = \lim_{N\to\infty} \alpha(N) = 0\,.$$

- generically, $\alpha(N)$ has the asymptotic behavior of the form

$$\alpha(N) = c_K + \sum_{D=0}^{K-1} \frac{c_D}{\left(\sqrt[K]{N}\right)^{K-D}} + \mathcal{O}\left(\frac{1}{N\,\sqrt[K]{N}}\right) + \mathcal{R}(N)\,,$$

- and if only one c_D coefficient is different from zero, the hypersurface has a well-defined notion of dimension

$$D = \lim_{N\to\infty} D(N) = \lim_{N\to\infty} K\left[1 + \frac{\log\alpha(N)}{\log N}\right]\,.$$

FIELD THEORY and SPACETIME

Extend the gedanken-experiment to include non-mechanical observables:

- consider a fluid in motion, such as a river flowing through a riverbed,
- introduce N small probes which measure K different observables:

$$\rho^m\,,\quad p\,,\quad T\,,\quad \rho^e\,,\quad E\,,\quad B\,,\quad \vec{E}\cdot\vec{B}\,,\quad \ldots$$

- let the probes scatter throughout the river, and randomly activate and transmit the measured values to a computer,
- the computer remembers the obtained values of the observables, but *does not remember* their order,

$$\begin{aligned} \rho^m &= \{\rho^m_1, \ldots, \rho^m_N\}\,, \\ p &= \{p_1, \ldots, p_N\}\,, \\ T &= \{T_1, \ldots, T_N\}\,, \\ &\vdots \end{aligned}$$

- construct K-tuples by pairing the observables using random permutations,
- have the computer memorize those points on a K-dimensional scatter "plot".

FIELD THEORY and SPACETIME

Then discover that:

- there exists a special permutation, which features the four properties of (1) existence, (2) self-reinforcement, (3) dimensionality, and (4) topology,
- call this special permutation $\mathcal{M}$, and discover that for all observables, all physical systems, and *all experiments ever done*, it turns out that

$$\dim \mathcal{M} = 4\,, \qquad \mathcal{M} \text{ has simply connected topology}$$

- finally, introduce a chart

$$f : \mathcal{M} \to \mathbb{R}^4\,, \qquad (\rho^m,\, p,\, \dots) \mapsto (t,x,y,z)$$

  and its inverse

$$f^{-1} : \mathbb{R}^4 \to \mathcal{M}\,, \qquad (t,x,y,z) \mapsto \Big(\, \rho^m(t,x,y,z)\,,\, p(t,x,y,z)\,,\, \dots \,\Big)$$

- and note that $\mathcal{M}$ features invariance with respect to $\mathit{Diff}(\mathbb{R}^4)$.

$\Rightarrow$ ***SPACETIME MANIFOLD !!!***

CONCLUSION

There exists a unique signal in experimental data of our gedanken-experiment, that corresponds to the existence of a spacetime manifold, 4-dimensional and simply connected.

This signal is of the same quality as the signal for the existence of atoms, electrons, EM-field, and other phenomena in physics (including the Pope in Rome). One can therefore argue that the spacetime manifold is equally objective and ontologically real as these other phenomena.

Further topics for discussion:

- what about quantum mechanics and non-commuting observables?
- what about extra dimensions of spacetime and experiments (not yet done) at the Planck scale?
- what about global symmetries and uniqueness of the special permutation graph?
- what about the smoothness structure of spacetime?
- what about the notion of "emergence of spacetime"?
- etc…

THANK YOU!