Odbrana doktorske disertacije:

Kurantovi algebroidi u bozonskoj teoriji struna (Courant algebroids in bosonic string theory)

Ilija Ivanišević

Mentor: dr Ljubica Davidović
Komisija: prof. dr Voja Radovanović, prof. dr Maja Burić, dr Bojan Nikolić

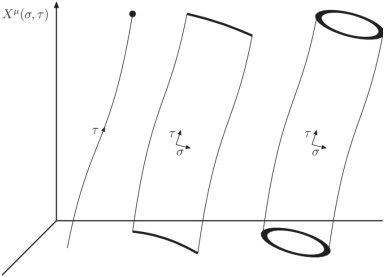
Grupa za gravitaciju, čestice i polja
Institut za fiziku u Beogradu

Publikacije

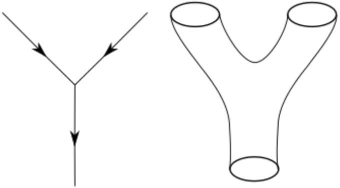
- I Ivanišević, Lj. Davidović, B Sazdović, *Courant bracket found out to be T-dual to Roytenberg bracket, Eur. Phys. J.* **C 80** (2020) 571.
- Lj. Davidović, I. Ivanišević, B. Sazdović, *Courant bracket as T-dual invariant extension of Lie bracket, JHEP* **03** (2021) 109.
- Lj. Davidović, I. Ivanišević, B. Sazdović, *Courant bracket twisted both by a 2-form B and by a bi-vector θ , Eur. Phys. J.* **C 81** (2021) 685.
- Lj. Davidović, I Ivanišević, B Sazdović, *Twisted C-Brackets, Fortschritte der Physik* **71** (2023) 2200187.

Teorija struna

- Teorija struna - osnovni elementi prirode su jednodimenzione strune. Različite čestice se manifestuju kao različite vibracije strune.
- Teorija predviđa bozon spina 2, odnosno **graviton**. Gravitacija koja se dobija teorijom struna je **renormalizabilna**.



svetska linija, svetska površ za otvorenu i svetska površ za zatvorenu strunu



Fajnmanov dijagram za čestične interakcije i za interakcije strune

- Bozonska teorija struna je prva razvijena.

Dejstvo za bozonsku strunu

Relativistička čestica

- Dejstvo za relativističku česticu je proporcionalno dužini svetske linije.

$$S_0 = m \int \sqrt{G_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}\, d\tau$$

- Kvadratni koren se može izbeći uvođenjem pomoćnog polja e

$$S = \frac{1}{2}\int d\tau \left(\frac{1}{e} G_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + m^2 e\right)$$

- Varijacija po x^μ:

$$\ddot{x}^\mu + \Gamma^\mu_{\nu\rho}\dot{x}^\nu \dot{x}^\rho = 0$$

Bozonska struna

- **Nambu-Gotovo dejstvo** je proporcionalno površini svetske površi

$$S_{NG} = \kappa \int_\Sigma d^2\xi \sqrt{-\det\Big(\partial_\alpha x^\mu \partial_\beta x^\nu\, G_{\mu\nu}\Big)}$$

- **Poljakovo dejstvo** se dobija uvođenjem metrike na svetskoj površi g

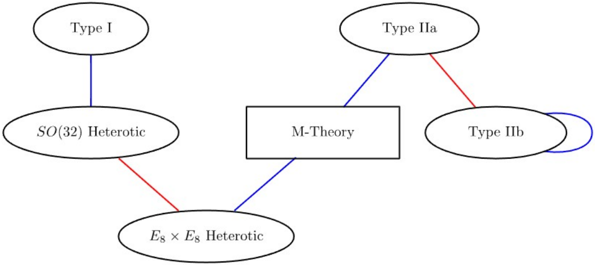
$$S_P = \frac{\kappa}{2}\int d^2\sigma \sqrt{-g}\; g^{\alpha\beta} G_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu$$

- Varijacija po x^μ:

$$\partial_+\partial_- x^\mu = 0\,, \quad \partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$$

Superstrune i M-teorija

- Fermioni su uvedeni supersimetrijom. Konstruisano je pet na prvi pogled različitih teorija superstruna bez anomalija.
- Edvard Viten je 1995. godine sugerisao da teorije nisu različite, već povezane mrežom dualnosti sa 11-dimenzionalnom M-teorijom.



Mreža dualnosti: crvene linije predstavljaju T-dualnost, dok su plave linije S-dualnost.

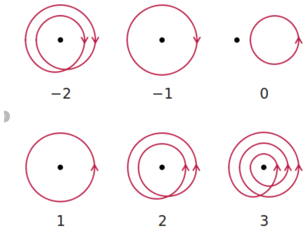
- **T-dualnost:** dualnost između teorija koje su definisane u različitim geometrijama.
- **S-dualnost:** dualnost teorija sa slabim i jakim konstantama kuplovanja.

T-dualnost- primer: bozonska struna u prostoru $\mathbb{R}^{1,24} \times S^1$

- Translacija duž kompaktifikovane dimenzije za a je generisana sa $e^{ip_{25}a}$. Ukoliko je $a = 2\pi R$, onda se mora dobiti jedinični operator, te se može uvesti **impulsni broj** n pomoću:

$$p^{25} = \frac{n}{R}\,, \quad n \in \mathbb{Z}\,.$$

- Oko kompaktifikovane dimenzije struna može da se obmotava. To se može karakterisati **brojem namotaja** m.



- T-dualnost - invarijantnost masenog spektra

$$M^2 = \frac{n^2}{R^2} + \frac{m^2 R^2}{l_s^4}\,, \quad R \leftrightarrow \frac{l_s^2}{R}\,, \quad m \leftrightarrow n$$

Lijev algebroid

- **Lijev algebroid** sastoji se od vektorskog raslojenja E, zagrade na glatkom preseku od E i projekcije na tangentno raslojenje (engl. *anchor*) ρ. Potrebno je da važe sledeći uslovi kompatibilnosti:

$$\rho[\xi_1,\xi_2] = [\rho(\xi_1),\rho(\xi_2)]_L\,,$$
$$[\xi_1, f\xi_2] = f[\xi_1,\xi_2] + (\mathcal{L}_{\rho(\xi_1)}f)\;\xi_2\,,$$
$$[\xi_1,[\xi_2,\xi_3]] + [\xi_2,[\xi_3,\xi_1]] + [\xi_3,[\xi_1,\xi_2]] = 0\,.$$

- Omogućava uvođenje poznatih geometrijskih koncepata za različita vektorska raslojenja.
- Lijev izvod:

$$\hat{\mathcal{L}}_\xi f = \mathcal{L}_{\rho(\xi)} f\,, \quad \hat{\mathcal{L}}_{\xi_1}\xi_2 = [\xi_1,\xi_2]\,.$$

- spoljašnji izvod:

$$\begin{aligned}\hat{d}\lambda(\xi_0,...,\xi_p) \quad &= \quad \sum_{i=0}^{p}(-1)^i\mathcal{L}_{\rho(\xi_i)}\Big(\lambda(\xi_0,...,\hat{\xi}_i,...,\xi_p)\Big)\\ &\quad + \sum_{i<j}(-1)^{i+j}\lambda([\xi_i,\xi_j],\xi_0,...,\hat{\xi}_i,...,\hat{\xi}_j,...,\xi_p)\,,\end{aligned}$$

Primer - Košulova zagrada

- Primer Lijevog algebroida: $\{T^\star\mathcal{M},\ [,]_\theta,\ \theta\}$
- Projekcija: $\theta(\lambda_1)\lambda_2 = \theta(\lambda_1,\lambda_2)\,, \quad \Big(\theta(\lambda_1)\Big)^\mu = \lambda_{1\nu}\theta^{\nu\mu}$
- Košulova zagrada

$$[\lambda_1,\lambda_2]_\theta = \mathcal{L}_{\theta(\lambda_1)}\lambda_2 - \mathcal{L}_{\theta(\lambda_2)}\lambda_1 - d(\theta(\lambda_1,\lambda_2))$$

- Poasonov uslov je neophodan da bi bio zadovoljen prvi uslov kompatibilnosti za Lijev algebroid

$$\theta([\lambda_1,\lambda_2]_\theta) = [\theta(\lambda_1),\theta(\lambda_2)]_L\,, \quad [\theta,\theta]_S = 0$$

$$[\theta,\theta]_S|^{\mu\nu\rho} = \theta^{\mu\sigma}\partial_\sigma\theta^{\nu\rho} + \theta^{\nu\sigma}\partial_\sigma\theta^{\rho\mu} + \theta^{\rho\sigma}\partial_\sigma\theta^{\mu\nu}$$

- Lijev izvod:

$$\hat{\mathcal{L}}_\lambda f = \mathcal{L}_{\theta(\lambda)} f\,, \quad \hat{\mathcal{L}}_{\lambda_1}\lambda_2 = [\lambda_1,\lambda_2]_\theta$$

- spoljašnji izvod:

$$(d_\theta f)^\mu = \theta^{\mu\nu}\partial_\nu f\,, \quad (d_\theta\xi)^{\mu\nu} = \theta^{\mu\rho}\partial_\rho\xi^\nu - \theta^{\nu\rho}\partial_\rho\xi^\mu - \xi^\rho\partial_\rho\theta^{\mu\nu}$$

Generalisana geometrija

- Generalisano tangentno raslojenje: $T\mathcal{M} \oplus T^\star\mathcal{M}$
- $O(D,D)$ invarijantni skalarni proizvod

$$\langle \Lambda_1, \Lambda_2 \rangle = \langle \xi_1 \oplus \lambda_1, \xi_2 \oplus \lambda_2 \rangle = i_{\xi_1}\lambda_2 + i_{\xi_2}\lambda_1$$

- Neka su data dva Lijeva algebroida na $T\mathcal{M}$ i $T^\star\mathcal{M}$ sa zagradama $[,]_L$ i $[,]_{L^\star}$. Može se konstruisati antisimetrična zagrada na generalisanom tangentnom raslojenju

$$\begin{aligned}[\Lambda_1, \Lambda_2] = \quad & \Big([\xi_1, \xi_2]_L + {\mathcal{L}^\star}_{\lambda_1}\xi_2 - {\mathcal{L}^\star}_{\lambda_2}\xi_1 - \frac{1}{2}d^\star(i_{\xi_1}\lambda_2 - i_{\xi_2}\lambda_1)) \\ & \oplus\Big([\lambda_1, \lambda_2]_{L^\star} + \mathcal{L}_{\xi_1}\lambda_2 - \mathcal{L}_{\xi_2}\lambda_1 - \frac{1}{2}d(i_{\xi_1}\lambda_2 - i_{\xi_2}\lambda_1)\Big)\end{aligned}$$

- Kurantova zagrada

$$[\Lambda_1, \Lambda_2]_{\mathcal{C}} = [\xi_1, \xi_2]_L \oplus \Big(\mathcal{L}_{\xi_1}\lambda_2 - \mathcal{L}_{\xi_2}\lambda_1 - \frac{1}{2}d(i_{\xi_1}\lambda_2 - i_{\xi_2}\lambda_1)\Big)$$

Kurantov algebroid

- Kurantov algebroid $\Big(E, \langle , \rangle, [,], \rho\Big)$. Može se definisati izvod $\langle \mathcal{D}f, \Lambda \rangle = \mathcal{L}_{\rho(\Lambda)} f$. Uslovi kompatibilnosti:

  1. $\rho[\Lambda_1, \Lambda_2] = [\rho(\Lambda_1), \rho(\Lambda_2)]_L$
  2. $[\Lambda_1, f\Lambda_2] = f[\Lambda_1, \Lambda_2] + (\mathcal{L}_{\rho(\Lambda_1)} f)\Lambda_2 - \frac{1}{2}\langle \Lambda_1, \Lambda_2 \rangle \mathcal{D}f$
  3. $\mathcal{L}_{\rho(\Lambda_1)}\langle \Lambda_2, \Lambda_3 \rangle = \langle [\Lambda_1, \Lambda_2] + \frac{1}{2}\mathcal{D}\langle \Lambda_1, \Lambda_2 \rangle, \Lambda_3 \rangle + \langle \Lambda_2, [\Lambda_1, \Lambda_3] + \frac{1}{2}\mathcal{D}\langle \Lambda_1, \Lambda_3 \rangle \rangle$
  4. $\langle \mathcal{D}f, \mathcal{D}g \rangle = 0$
  5. $\text{Jac}(\Lambda_1, \Lambda_2, \Lambda_3) = \mathcal{D}\text{Nij}(\Lambda_1, \Lambda_2, \Lambda_3)$

- Jakobiator

$$\text{Jac}(\Lambda_1, \Lambda_2, \Lambda_3) = [[\Lambda_1, \Lambda_2], \Lambda_3] + [[\Lambda_2, \Lambda_3], \Lambda_1] + [[\Lambda_3, \Lambda_1], \Lambda_2]$$

- Nejenhujsev operator

$$\text{Nij}(\Lambda_1, \Lambda_2, \Lambda_3) = \frac{1}{6}\Big(\langle [\Lambda_1, \Lambda_2], \Lambda_3 \rangle + \langle [\Lambda_2, \Lambda_3], \Lambda_1 \rangle + \langle [\Lambda_3, \Lambda_1], \Lambda_2 \rangle\Big)$$

Standardni Kurantov algebroid

- **Standardni Kurantov algebroid** sastoji se od generalisanog tangentnog raslojenja kao vektorskog raslojenja $E = T\mathcal{M} \oplus T^\star\mathcal{M}$, Kurantove zagrade kao zagrade, $O(D,D)$ invarijantnog simetričnog skalarnog proizvoda i projekcije $\pi(\xi \oplus \lambda) = \xi$ kao projekcije (anchor)
- Prvi uslov - Kurantova zagrada postaje Lijeva zagrada na tangentnom raslojenju

$$\pi([\Lambda_1, \Lambda_2]_{\mathcal{C}}) = [\pi(\Lambda_1), \pi(\Lambda_2)]_{\mathcal{C}} = [\xi_1, \xi_2]_L$$

- Kurantova zagrada **ne zadovoljava** Lajbnicovo pravilo

$$[\Lambda_1, f\Lambda_2]_{\mathcal{C}} = f[\Lambda_1, \Lambda_2]_{\mathcal{C}} + (\mathcal{L}_{\pi(\Lambda_1)} f)\ \Lambda_2 - f\frac{1}{2} d\langle \Lambda_1, \Lambda_2 \rangle$$

- Kurantova zagrada **ne zadovoljava** Jakobijev identitet

$$\text{Jac}(\Lambda_1, \Lambda_2, \Lambda_3) = d\text{Nij}(\Lambda_1, \Lambda_2, \Lambda_3)$$

$$\begin{aligned}
\text{Jac}(\Lambda_1, \Lambda_2, \Lambda_3) &= [[\Lambda_1, \Lambda_2]_{\mathcal{C}}, \Lambda_3]_{\mathcal{C}} + [[\Lambda_2, \Lambda_3]_{\mathcal{C}}, \Lambda_1]_{\mathcal{C}} + [[\Lambda_3, \Lambda_1]_{\mathcal{C}}, \Lambda_2]_{\mathcal{C}} \\
\text{Nij}(\Lambda_1, \Lambda_2, \Lambda_3) &= \frac{1}{6}\Big(\langle [\Lambda_1, \Lambda_2]_{\mathcal{C}}, \Lambda_3 \rangle + \langle [\Lambda_2, \Lambda_3]_{\mathcal{C}}, \Lambda_1 \rangle + \langle [\Lambda_3, \Lambda_1]_{\mathcal{C}}, \Lambda_2 \rangle\Big)
\end{aligned}$$

Dirakove strukture

- Dirakove strukture predstavljaju podraslojenja maksimalne dimenzije koja su izotropna u odnosu na skalarni proizvod i čiji preseci su zatvoreni u odnosu na zagradu Kurantovog algebroida. Na njima Kurantov algebroid postaje Lijev algebroid.
- Izotropnost: $\langle \Lambda_1, \Lambda_2 \rangle = 0$
- Dirakove strukture mogu imati sledeće oblike:

$$\mathcal{V}_B(\Lambda) = \xi^\mu \oplus 2B_{\mu\nu}\xi^\nu \,, \quad \mathcal{V}_\theta(\Lambda) = \kappa\theta^{\mu\nu}\lambda_\nu \oplus \lambda_\mu \,.$$

- **Primer** - Standardni Kurantov algebroid
- Simplektičke mnogostrukosti

$$[\mathcal{V}_B(\Lambda_1), \mathcal{V}_B(\Lambda_2)]_{\mathcal{C}} = \mathcal{V}_B([\Lambda_1, \Lambda_2]_{\mathcal{C}}) \,, \quad dB = 0$$

- Poasonove mnogostrukosti

$$[\mathcal{V}_\theta(\Lambda_1), \mathcal{V}_\theta(\Lambda_2)]_{\mathcal{C}} = \mathcal{V}_\theta([\Lambda_1, \Lambda_2]_{\mathcal{C}}) \,, \quad [\theta, \theta]_S = 0$$

Bozonski σ-model

- Dejstvo

$$\mathcal{S} = \kappa \int_{\Sigma} d\sigma d\tau \Big[B_{\mu\nu}(x) + \frac{1}{2} G_{\mu\nu}(x) \Big] \partial_+ x^\mu \partial_- x^\nu \,,$$

- Kanonski impuls

$$\pi_\mu = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} = \kappa G_{\mu\nu} \dot{x}^\nu - 2\kappa B_{\mu\nu} x'^\nu \,.$$

- Hamiltonijan

$$\mathcal{H}_C = \pi_\mu \dot{x}^\mu - \mathcal{L} = \frac{1}{2\kappa} \pi_\mu (G^{-1})^{\mu\nu} \pi_\nu + \frac{\kappa}{2} x'^\mu G^E_{\mu\nu} x'^\nu - 2 x'^\mu B_{\mu\rho} (G^{-1})^{\rho\nu} \pi_\nu$$
$$G_E^{\mu\nu} = G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}$$

- Matrični oblik Hamiltonijana

$$\mathcal{H}_C = \frac{1}{2\kappa} (X^T)^M H_{MN} X^N$$
$$H_{MN} = \begin{pmatrix} G^E_{\mu\nu} & 2(BG^{-1})_\mu{}^\nu \\ -2(G^{-1}B)^\mu{}_\nu & (G^{-1})^{\mu\nu} \end{pmatrix} , \quad X^M = \begin{pmatrix} \kappa x'^\mu \\ \pi_\mu \end{pmatrix} .$$

Simetrije bozonske zatvorene strune

- Simetrije su opisane generatorima $\mathcal{H}_{(G,B)} + \{\mathcal{G}, \mathcal{H}_{(G,B)}\} = \mathcal{H}_{(G+\delta G, B+\delta B)}$
- Difeomorfizmi:

$$\mathcal{G}_{\xi} = \int_0^{2\pi} d\sigma \xi^{\mu} \pi_{\mu}$$

$$\begin{aligned} \delta_{\xi} G_{\mu\nu} &= \mathcal{L}_{\xi} G_{\mu\nu} = \xi^{\rho}\partial_{\rho} G_{\mu\nu} + \partial_{\mu}\xi^{\rho} G_{\rho\nu} + \partial_{\nu}\xi^{\rho} G_{\rho\mu} \,, \\ \delta_{\xi} B_{\mu\nu} &= \mathcal{L}_{\xi} B_{\mu\nu} = \xi^{\rho}\partial_{\rho} B_{\mu\nu} - \partial_{\mu}\xi^{\rho} B_{\rho\nu} + B_{\mu\rho}\partial_{\nu}\xi^{\rho} \,. \end{aligned}$$

- Algebra difeomorfizama

$$\left\{\mathcal{G}_{\xi_1}, \mathcal{G}_{\xi_2}\right\} = -\mathcal{G}_{[\xi_1,\xi_2]_L} \,.$$

- Lokalne gradijentne transformacije:

$$\mathcal{G}_{\lambda} = \int_0^{2\pi} d\sigma \lambda_{\mu} \kappa x'^{\mu}$$

$$\begin{aligned} &\delta_{\lambda} G_{\mu\nu} = 0 \\ &\delta_{\lambda} B_{\mu\nu} = (d\lambda)_{\mu\nu} = \partial_{\mu}\lambda_{\nu} - \partial_{\nu}\lambda_{\mu} \end{aligned}$$

Simetrije bozonske zatvorene strune

- T-dualnost povezuje brojeve impulsa i brojeve namotaja.

$$P^\mu = \int d\sigma \pi_\mu \,, \quad W^\mu = \int d\sigma \kappa x'^\mu$$

- Dva generatora simetrija su međusobno T-dualna

$$\kappa x'^\mu \cong \pi_\mu \,.$$

- To nam je motivacija da uzmemo u obzir objedinjeni generator:

$$\mathcal{G}_\Lambda = \mathcal{G}_\xi + \mathcal{G}_\lambda = \int d\sigma \langle \Lambda, X \rangle$$

$$\Lambda^M = \begin{pmatrix} \xi^\mu \\ \lambda_\mu \end{pmatrix} \,, \quad X^M = \begin{pmatrix} \kappa x'^\mu \\ \pi_\mu \end{pmatrix}$$

- Algebra generatora zatvara se na Kurantovoj zagradi:

$$\left\{ \mathcal{G}_{\Lambda_1}, \mathcal{G}_{\Lambda_2} \right\} = -\mathcal{G}_{[\Lambda_1, \Lambda_2]_\mathcal{C}}$$

- Kurantova zagrada je ekstenzija Lijeve zagrade koja je invarijantna na T-dualnost.

Zavrnuta Kurantova zagrada

- Transformacije koje čine skalarni proizvod invarijantan čine $O(D,D)$ grupu simetrija
- Metoda za određivanje zavrnute Kurantove zagrade:

$$\hat{X}^M = (e^T)^M_{\ N} X^N\,, \quad \hat{\Lambda}^M = (e^T)^M_{\ N} \Lambda^N$$

$$\mathcal{G}_\Lambda = \int d\sigma \langle \Lambda, X\rangle = \int d\sigma \langle e^T\Lambda, e^T X\rangle = \int d\sigma \langle \hat{\Lambda}, \hat{X}\rangle = \mathcal{G}^{(T)}_{\hat{\Lambda}}$$

- Algebra - zavrnuta Kurantova zagrada:

$$\left\{\mathcal{G}^{(T)}_{\hat{\Lambda}_1}, \mathcal{G}^{(T)}_{\hat{\Lambda}_2}\right\} = -\mathcal{G}^{(T)}_{[\hat{\Lambda}_1,\hat{\Lambda}_2]_{\mathcal{C}_T}}\,, \quad [\hat{\Lambda}_1,\hat{\Lambda}_2]_{\mathcal{C}_T} = e^T[e^{-T}\hat{\Lambda}_1, e^{-T}\hat{\Lambda}_2]_{\mathcal{C}}$$

- Uslovi preslikavanja projekcije i izvoda

$$\rho = \pi \circ e^{-T}\,, \quad \mathcal{D}f = e^T df$$

- Svih pet uslova kompatibilnosti između elemenata Kurantovog algebroida su zadovoljeni.
- Zavrnute Kurantove zagrade mogu da opisuju flukseve

B-zavrnuta Kurantova zagrada

- B-transformacije $e^{\hat{B}}$

$$\hat{B} = \begin{pmatrix} 0 & 0 \\ 2B & 0 \end{pmatrix} , \quad e^{\hat{B}} = \begin{pmatrix} 1 & 0 \\ 2B & 1 \end{pmatrix}$$

- Hamiltonijan se može napisati u dijagonalnoj formi

$$\mathcal{H}_{\mathcal{C}} = \frac{1}{2\kappa}(X^T)^M H_{MN} X^N = \frac{1}{2\kappa}\hat{X}^M G_{MN} \hat{X}^N$$

$$\hat{X}^M = (e^{\hat{B}})^M{}_N X^N = \begin{pmatrix} \kappa x'^\mu \\ \pi_\mu + 2\kappa B_{\mu\nu} x'^\nu \end{pmatrix} \equiv \begin{pmatrix} \kappa x'^\mu \\ i_\mu \end{pmatrix} , \quad G_{MN} = \begin{pmatrix} G_{\mu\nu} & 0 \\ 0 & (G^{-1})^{\mu\nu} \end{pmatrix}$$

- Struje i_μ:

$$\{i_\mu(\sigma), i_\nu(\bar{\sigma})\} = -2\kappa B_{\mu\nu\rho} x'^\rho \delta(\sigma - \bar{\sigma}) , \quad B_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$$

B-zavrnuta Kurantova zagrada

- B-zavrnuta Kurantova zagrada

$$[\Lambda_1,\Lambda_2]_{\mathcal{C}_B}=[\xi_1,\xi_2]_L\oplus\Big(\mathcal{L}_{\xi_1}\lambda_2-\mathcal{L}_{\xi_2}\lambda_1-\frac{1}{2}d(i_{\xi_1}\lambda_2-i_{\xi_2}\lambda_1)+dB\Big)$$

- Dirakove strukture:

$$\Big[\mathcal{V}_B(\Lambda_1),\mathcal{V}_B(\Lambda_2)\Big]_{\mathcal{C}_B}=\mathcal{V}_B\Big([\Lambda_1,\Lambda_2]_{\mathcal{C}_B}\Big)\,,\quad\forall\ dB$$

$$[\mathcal{V}_\theta(\Lambda_1),\mathcal{V}_\theta(\Lambda_2)]_{\mathcal{C}_B}=\mathcal{V}_\theta\Big([\Lambda_1,\Lambda_2]_{\mathcal{C}_B}\Big)\,,\quad\mathcal{R}=\frac{1}{2}[\theta,\theta]_S+2\kappa\wedge^3\theta\ dB=0$$

θ-zavrnuta Kurantova zagrada

- T-dualna polja

$$^\star G^{\mu\nu} = (G_E^{-1})^{\mu\nu}\,, \quad ^\star B^{\mu\nu} = \frac{\kappa}{2}\theta^{\mu\nu}$$

$$G^E_{\mu\nu} = G_{\mu\nu} - 4(BG^{-1}B)_{\mu\nu}\,, \quad \theta^{\mu\nu} = \frac{2}{\kappa}(G_E^{-1}BG^{-1})^{\mu\nu}$$

- θ-transformacija $e^{\hat\theta}$

$$\hat\theta = \begin{pmatrix} 0 & \kappa\theta \\ 0 & 0 \end{pmatrix}\,, \quad e^{\hat\theta} = \begin{pmatrix} 1 & \kappa\theta \\ 0 & 1 \end{pmatrix}$$

- Hamiltonijan se može zapisati u dijagonalnoj formi:

$$^\star\hat{\mathcal{H}}_C = \frac{1}{2\kappa}\hat X^M {}^\star G_{MN}\hat X^N\,, \quad \hat X^M = \begin{pmatrix} \kappa x'^\mu + \kappa\theta^{\mu\nu}\pi_\nu \\ \pi_\mu \end{pmatrix} \equiv \begin{pmatrix} k^\mu \\ \pi_\mu \end{pmatrix}$$

$$^\star G_{MN} = \begin{pmatrix} ^\star(G^{-1})_{\mu\nu} & 0 \\ 0 & ^\star G^{\mu\nu} \end{pmatrix} = \begin{pmatrix} G^E_{\mu\nu} & 0 \\ 0 & (G_E^{-1})^{\mu\nu} \end{pmatrix}$$

- Struje k^μ:

$$\{k^\mu(\sigma), k^\nu(\bar\sigma)\} = -\kappa Q_\rho{}^{\mu\nu} k^\rho \delta(\sigma-\bar\sigma) - \kappa^2 R^{\mu\nu\rho}\pi_\rho\delta(\sigma-\bar\sigma)$$

$$Q_\rho{}^{\mu\nu} = \partial_\rho\theta^{\mu\nu}, \qquad R^{\mu\nu\rho} = \theta^{\mu\sigma}\partial_\sigma\theta^{\nu\rho} + \theta^{\nu\sigma}\partial_\sigma\theta^{\rho\mu} + \theta^{\rho\sigma}\partial_\sigma\theta^{\mu\nu}$$

θ-zavrnuta Kurantova zagrada

- θ-zavrnuta Kurantova zagrada

$$\begin{aligned} \xi &= [\xi_1,\xi_2]_L - \kappa[\xi_2,\lambda_1\theta]_L + \kappa[\xi_1,\lambda_2\theta]_L + \frac{\kappa^2}{2}[\theta,\theta]_S(\lambda_1,\lambda_2,.) \\ &\quad -\kappa\theta\Big(\mathcal{L}_{\xi_2}\lambda_1 - \mathcal{L}_{\xi_1}\lambda_2 + \frac{1}{2}d(i_{\xi_1}\lambda_2 - i_{\xi_2}\lambda_1)\Big) \\ \lambda &= \mathcal{L}_{\xi_1}\lambda_2 - \mathcal{L}_{\xi_2}\lambda_1 - \frac{1}{2}d(i_{\xi_1}\lambda_2 - i_{\xi_2}\lambda_1) + \kappa[\lambda_1,\lambda_2]_\theta \end{aligned}$$

- Dirakove strukture

$$[\mathcal{V}_B(\Lambda_1),\mathcal{V}_B(\Lambda_2)]_{\mathcal{C}_\theta} = \mathcal{V}_B[\Lambda_1,\Lambda_2]_{\mathcal{C}_\theta}\,, \quad d(BG^{-1}G_E) = 0$$
$$[\mathcal{V}_\theta(\Lambda_1),\mathcal{V}_\theta(\Lambda_2)]_{\mathcal{C}_\theta} = 0\,, \quad \forall\theta$$

θ-zavrnuta Kurantova zagrada

- Ukoliko se izvrši zamena veličina sa njihovim T-dualima, bazis u kome je definisan generator koji daje B-zavrnutu Kurantovu zagradu daje bazis koji daje θ-zavrnutu Kurantovu zagradu

$$\pi_\mu \leftrightarrow \kappa x'^\mu , \quad 2B_{\mu\nu} \leftrightarrow \kappa\theta^{\mu\nu}$$
$$i_\mu = \pi_\mu + 2\kappa B_{\mu\nu} x'^\nu \leftrightarrow \kappa x'^\mu + \kappa\theta^{\mu\nu}\pi_\nu = k^\mu$$

- Određen je izomorfizam između ovih Kurantovih algebroida koji odgovara T-dualnim transformacijama

$$\varphi = (e^{\hat{\theta}} e^{-\hat{B}})^M{}_N = \begin{pmatrix} \delta^\mu_\nu - 2\kappa(\theta B)^\mu{}_\nu & \kappa\theta^{\mu\nu} \\ -2B_{\mu\nu} & \delta^\nu_\mu \end{pmatrix} .$$

$$\varphi[\Lambda_1, \Lambda_2]_{\mathcal{C}_B} = \Big([\varphi(\Lambda_1), \varphi(\Lambda_2)]_{\mathcal{C}_\theta} \Big) .$$

B-θ-zavrnuta Kurantova zagrada

- Da li postoji zavrnuta Kurantova zagrada koja sadrži sve flukseve koja je pri tome invarijantna na T-dualnost?
- Prva ideja: zavrtanje učiniti najpre poljem B, pa poljem θ. Problem - ove dve transformacije ne komutiraju i zbog toga struje koje se dobiju neće biti međusobno T-dualne.
- Druga ideja: konstrukcija matrice $e^{\check{B}}$

$$\check{B} = \hat{B} + \hat{\theta} = \begin{pmatrix} 0 & \kappa\theta \\ 2B & 0 \end{pmatrix}$$

$$e^{\check{B}} = \begin{pmatrix} \mathcal{C} & \kappa\mathcal{S}\theta \\ 2B\mathcal{S} & \mathcal{C}^T \end{pmatrix}, \quad \mathcal{C} = \cosh(\sqrt{\alpha}), \quad \mathcal{S} = \frac{\sinh(\sqrt{\alpha})}{\sqrt{\alpha}}, \quad \alpha = 2\kappa\theta B$$

- Struje su međusobno T-dualne

$$\check{X}^M = (e^{\check{B}})^M{}_N X^N = \begin{pmatrix} \check{k}^\mu \\ \check{\imath}_\mu \end{pmatrix},$$

$$\check{k}^\mu = \kappa\mathcal{C}^\mu{}_\nu x'^\nu + \kappa(\mathcal{S}\theta)^{\mu\nu}\pi_\nu,$$

$$\check{\imath}_\mu = 2(B\mathcal{S})_{\mu\nu}x'^\nu + (\mathcal{C}^T)_\mu{}^\nu\pi_\nu.$$

$$2B_{\mu\nu} \leftrightarrow \kappa\theta^{\mu\nu}, \pi_\mu \leftrightarrow \kappa x'^\mu$$

$$\alpha^\mu{}_\nu \leftrightarrow (\alpha^T)_\nu{}^\mu$$

$$\mathcal{C} \leftrightarrow \mathcal{C}^T, \quad \mathcal{S} \leftrightarrow \mathcal{S}^T$$

$$\check{k}^\mu \leftrightarrow \check{\imath}_\mu$$

B-θ-zavrnuta Kurantova zagrada

- Algebra struja:

$$\{\breve{\imath}_\mu(\sigma), \breve{\imath}_\nu(\bar{\sigma})\} = -2\breve{\mathcal{B}}_{\mu\nu\rho}\breve{k}^\rho \delta(\sigma-\bar{\sigma}) - \breve{\mathcal{F}}_{\mu\nu}{}^\rho\, \breve{\imath}_\rho \delta(\sigma-\bar{\sigma})$$
$$\{\breve{k}^\mu(\sigma), \breve{k}^\nu(\bar{\sigma})\} = -\kappa \breve{\mathcal{Q}}_\rho{}^{\mu\nu}\breve{k}^\rho \delta(\sigma-\bar{\sigma}) - \kappa^2 \breve{\mathcal{R}}^{\mu\nu\rho}\breve{\imath}_\rho \delta(\sigma-\bar{\sigma})$$
$$\{\breve{\imath}_\mu(\sigma), \breve{k}^\nu(\bar{\sigma})\} = \kappa\delta_\mu^\nu \delta'(\sigma-\bar{\sigma}) + \breve{\mathcal{F}}_{\mu\rho}{}^\nu\, \breve{k}^\rho \delta(\sigma-\bar{\sigma}) - \kappa \breve{\mathcal{Q}}_\mu{}^{\nu\rho}\breve{\imath}_\rho \delta(\sigma-\bar{\sigma})$$

- Fluksevi:

$$\breve{\mathcal{B}}_{\mu\nu\rho} = \mathcal{C}^\alpha{}_\mu \mathcal{C}^\beta{}_\nu \mathcal{C}^\gamma{}_\rho (\partial_\alpha \breve{B}_{\beta\gamma} + \partial_\beta \breve{B}_{\gamma\alpha} + \partial_\gamma \breve{B}_{\alpha\beta})$$
$$\breve{\mathcal{F}}_{\mu\nu}{}^\rho = \breve{f}_{\mu\nu}{}^\rho - 2\kappa \breve{\mathcal{B}}_{\mu\nu\sigma}\breve{\theta}^{\sigma\rho}\,, \quad \breve{f}_{\mu\nu}{}^\rho = (\mathcal{C}^{-1})^\rho{}_\sigma \left(\hat{\partial}_\mu \mathcal{C}^\sigma{}_\nu - \hat{\partial}_\nu \mathcal{C}^\sigma{}_\mu\right)$$
$$\breve{\mathcal{Q}}_\rho{}^{\mu\nu} = \breve{Q}_\rho{}^{\mu\nu} + 2\kappa \breve{\theta}^{\mu\alpha}\breve{\theta}^{\nu\beta}\breve{\mathcal{B}}_{\rho\alpha\beta}\,, \quad \breve{Q}_\rho{}^{\mu\nu} = \hat{\partial}_\rho \breve{\theta}^{\mu\nu} + \breve{f}_{\rho\sigma}{}^\mu\, \breve{\theta}^{\sigma\nu} - \breve{f}_{\rho\sigma}{}^\nu\, \breve{\theta}^{\sigma\mu}$$
$$\breve{\mathcal{R}}^{\mu\nu\rho} = \breve{R}^{\mu\nu\rho} + 2\kappa \breve{\theta}^{\mu\alpha}\breve{\theta}^{\nu\beta}\breve{\theta}^{\rho\gamma}\breve{\mathcal{B}}_{\alpha\beta\gamma}\,, \quad \breve{R}^{\mu\nu} = \breve{\theta}^{\mu\sigma}\hat{\partial}_\sigma \breve{\theta}^{\nu\rho} - \breve{\theta}^{\mu\alpha}\breve{\theta}^{\rho\beta}\breve{f}_{\alpha\beta}{}^\nu + cyclic$$

- Efektivna polja:

$$\breve{B}_{\mu\nu} = (B\mathcal{S}\mathcal{C}^{-1})_{\mu\nu}\,, \quad \breve{\theta}^{\mu\nu} = (\mathcal{S}\mathcal{C}^{-1}\theta)^{\mu\nu}\,, \quad \hat{\partial}_\mu = (\mathcal{C}^T)_\mu{}^\nu \partial_\nu$$

$B-\theta$-zavrnuta Kurantova zagrada

- Zagrada:

$$
\begin{aligned}
\xi &= [\xi_1, \xi_2]_{\hat{L}} - \kappa\breve{\theta}\Big(\hat{\mathcal{L}}_{\xi_1}\lambda_2 - \hat{\mathcal{L}}_{\xi_2}\lambda_1 - \frac{1}{2}\hat{d}(i_{\xi_1}\lambda_2 - i_{\xi_2}\lambda_1)\Big) \\
&+[\xi_1, \kappa\breve{\theta}(\lambda_2)]_{\hat{L}} - [\xi_2, \kappa\breve{\theta}(\lambda_1)]_{\hat{L}} + \frac{\kappa^2}{2}[\breve{\theta}, \breve{\theta}]_{\hat{S}}(\lambda_1, \lambda_2, .) \\
&+2\kappa\breve{\theta}\ \hat{d}\hat{B}(., \xi_1, \xi_2) - 2\wedge^2 \kappa\breve{\theta}\ \hat{d}\hat{B}(., \lambda_1, \xi_2) + 2\wedge^2 \kappa\breve{\theta}\ \hat{d}\hat{B}(., \lambda_2, \xi_1) + 2\wedge^3 \kappa\breve{\theta}\ \hat{d}\hat{B}(\lambda_1, \lambda_2, .) \\
\lambda &= \hat{\mathcal{L}}_{\xi_1}\lambda_2 - \hat{\mathcal{L}}_{\xi_2}\lambda_1 + \frac{1}{2}\hat{d}(i_{\xi_1}\lambda_2 - i_{\xi_2}\lambda_1) + \kappa[\lambda_1, \lambda_2]_{\breve{\theta}} \\
&+2\hat{d}\hat{B}(\xi_1, \xi_2, .) - 2\kappa\breve{\theta}\ \hat{d}\hat{B}(\lambda_2, ., \xi_1) + 2\kappa\breve{\theta}\ \hat{d}\hat{B}(\lambda_1, ., \xi_2) + 2\wedge^2 \kappa\breve{\theta}\ \hat{d}\hat{B}(\lambda_1, \lambda_2, .)
\end{aligned}
$$

- Zavrnuta Lijeva zagrada: $[\xi_1, \xi_2]_{\hat{L}} = \mathcal{C}^{-1}[\mathcal{C}\xi_1, \mathcal{C}\xi_2]_L$
- Zavrnuta Košulova zagrada: $[\lambda_1, \lambda_2]_{\breve{\theta}} = \hat{\mathcal{L}}_{\breve{\theta}(\lambda_1)}\lambda_2 - \hat{\mathcal{L}}_{\breve{\theta}(\lambda_2)}\lambda_1 - \hat{d}(\breve{\theta}(\lambda_1, \lambda_2))$
- Zavrnuta Šuten-Nejenhuiseva zagrada $[\breve{\theta}, \breve{\theta}]_{\hat{S}} = \hat{d}_{\breve{\theta}}\breve{\theta}$
- Fluksevi na Dirakovim strukturama mogu da postoje bez ograničenja.

Dupla teorija

- U duploj teoriji, fazni prostor je dupliran. Sva polja zavise od početnih koordinata x^μ i od T-dualnih koordinata y_μ.
- Lagranžijan

$$\mathcal{L} = \frac{\kappa}{2} \partial_+ X^M H_{MN} \partial_- X^N \,, \quad X^M = \begin{pmatrix} x^\mu \\ y_\mu \end{pmatrix}$$

$$H_{MN} = \begin{pmatrix} G^E_{\mu\nu}(x,y) & -2B_{\mu\rho}(x,y)(G^{-1})^{\rho\nu}(x,y) \\ 2(G^{-1})^{\mu\rho}(x,y)B_{\rho\nu}(x,y) & (G^{-1})^{\mu\nu}(x,y) \end{pmatrix} .$$

- T-dualnost je simetrija dejstva

$$\partial_\pm X^M \cong \pm \eta^{MN} H_{NK} \, \partial_\pm X^K \,, \quad \eta^{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- Hamiltonijan

$$\mathcal{H}_C = \Pi_M \dot{X}^M - \mathcal{L} = \frac{1}{2\kappa} \Pi_M H^{MN} \Pi_N + \frac{\kappa}{2} X'^M H_{MN} X'^N \,, \quad \Pi^M = \begin{pmatrix} \pi_\mu \\ {}^\star\pi^\mu \end{pmatrix}$$

$$\pi_\mu = G^E_{\mu\nu} \dot{x}^\nu - 2(BG^{-1})_\mu{}^\nu \dot{y}_\nu \,,$$

$${}^\star\pi^\mu = (G^{-1})^{\mu\nu} \dot{y}_\nu + 2(G^{-1}B)^\mu{}_\nu \dot{x}^\nu$$

C-zagrada

- Generator difeomorfizama i T-dualnih difeomorfizama

$$\mathcal{G}_\Lambda = \int d\sigma \langle \Lambda, \Pi \rangle \,, \quad \Lambda^M(X) = \begin{pmatrix} \xi^\mu(x,y) \\ \lambda_\mu(x,y) \end{pmatrix}$$

- C-zagrada

$$\{\mathcal{G}_{\Lambda_1}, \mathcal{G}_{\Lambda_2}\} = -\mathcal{G}_{[\Lambda_1,\Lambda_2]_{\mathbf{C}}} \,,$$

$$\left([\Lambda_1,\Lambda_2]_{\mathbf{C}}\right)^M = \Lambda_1^N \partial_N \Lambda_2^M - \Lambda_2^N \partial_N \Lambda_1^M - \frac{1}{2}\left(\Lambda_1^N \partial^M \Lambda_{2N} - \Lambda_2^N \partial^M \Lambda_{1N}\right)$$

$$\partial_M = \begin{pmatrix} \partial_\mu \\ \tilde{\partial}^\mu \end{pmatrix} \,, \qquad \left(\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \,, \quad \tilde{\partial}^\mu \equiv \frac{\partial}{\partial y_\mu}\right)$$

- C-zagrada je uopštenje Lijeve zagrade na dupli prostor. Moguće je definisati uopšteni Lijev izvod koji deluje na sve indekse kao da su i kontravarijantni i kovarijantni

$$\hat{\mathcal{L}}_\Lambda H^{MN} = \Lambda^P \partial_P H^{MN} + (\partial^M \Lambda_P - \partial_P \Lambda^M) H^{PN} + (\partial^N \Lambda_P - \partial_P \Lambda^N) H^{MP} \,.$$

- Projekcijom C-zagrade na početni (ili T-dualni) D-dimenzionalni potprostor, dobijamo Kurantovu zagradu.

B-zavrnuta C-zagrada

- Definisali smo B-zavrnutu C-zagradu po analogiji sa Kurantovom zagradom

$$[\Lambda_1, \Lambda_2]_{\mathbf{C}_B} = e^{\hat{B}}[e^{-\hat{B}}\Lambda_1, e^{-\hat{B}}\Lambda_2]_{\mathbf{C}} \,.$$

- Pošto se i u duploj teoriji generator može napisati kao skalarni proizvod, može se primeniti naš metod računanja zavrnutih zagrada.
- B-zavrnuta C-zagrada

$$\Big([\Lambda_1, \Lambda_2]_{\mathbf{C}_B}\Big)^M = \Lambda_1^N \hat{\partial}_N \Lambda_2^M - \Lambda_2^N \hat{\partial}_N \Lambda_1^M - \frac{1}{2}\Big(\Lambda_1^N \hat{\partial}^M \Lambda_{2N} - \Lambda_2^N \hat{\partial}^M \Lambda_{1N}\Big) + \Lambda_{1N}\Lambda_{2Q}\hat{B}^{MNQ}$$

$$\hat{B}^{MNQ} = \hat{\partial}^M \hat{B}^{NQ} + \hat{\partial}^N \hat{B}^{QM} + \hat{\partial}^Q \hat{B}^{MN} \,, \quad \hat{\partial}^M = (e^{\hat{B}})^M{}_K \partial^K = \partial^M + \hat{B}^M_{\ K}\, \partial^K \,.$$

- Projekcijom na **početni prostor ($y = 0$)** dobija se B-**zavrnuta Kurantova zagrada**, dok se projekcijom na **T-dualni prostor ($x = 0$)** dobija θ-**zavrnuta Kurantova zagrada**.

θ-zavrnuta C-zagrada

- Definisali smo θ-zavrnutu C-zagradu po analogiji sa Kurantovom zagradom

$$[\Lambda_1, \Lambda_2]_{\mathbf{C}_\theta} = e^{\hat{\theta}}[e^{-\hat{\theta}}\Lambda_1, e^{-\hat{\theta}}\Lambda_2]_{\mathbf{C}} .$$

- θ-zavrnuta C-zagrada

$$\left([\Lambda_1, \Lambda_2]_{\mathbf{C}_\theta}\right)^M = \Lambda_1^N \hat{\partial}_N \Lambda_2^M - \Lambda_2^N \hat{\partial}_N \Lambda_1^M - \frac{1}{2}\left(\Lambda_1^N \hat{\partial}^M \Lambda_{2N} - \Lambda_2^N \hat{\partial}^M \Lambda_{1N}\right) + \Lambda_{1N}\Lambda_{2Q}\hat{\theta}^{MNQ}$$

$$\hat{\theta}^{MNQ} = \hat{\partial}^M \hat{\theta}^{NQ} + \hat{\partial}^N \hat{\theta}^{QM} + \hat{\partial}^Q \hat{\theta}^{MN} , \quad \hat{\partial}^M = (e^{\hat{\theta}})^M{}_K \partial^K = \partial^M + \hat{\theta}^M_{\ K}\, \partial^K .$$

- Projekcijom na **početni prostor ($y = 0$)** dobija se **θ-zavrnuta Kurantova zagrada**, dok se projekcijom na **T-dualni fazni prostor ($x = 0$)** dobija ***B*-zavrnuta Kurantova zagrada**

Zaključak

1. Kurantova zagrada je ekstenzija Lijeve zagrade invarijantna na T-dualnost.
2. Zavrtanje Kurantove zagrade poljem B dovodi do H-fluksa, dok zavrtanje poljem θ dovodi do Q i R fluksa. Ove zagrade su povezane T-dualnim transformacijama.
3. Razvijena je metoda određivanja zavrnutih Kurantovih zagrada i njima odgovarajućih Kurantovih algebroida.
4. Određena je zavrnuta Kurantova zagrada sa svim fluksevima koja je pri tom i invarijantna na T-dualne transformacije.
5. Uopšteni su rezultati za duplu teoriju, gde je pokazano da algebra simetrija postaje C-zagrada.
6. Određene su zavrnute C-zagrade i pokazano je da u međusobno T-dualnim faznim potprostorima postaju međusobno T-dualne zavrnute Kurantove zagrade.

Hvala na pažnji