T-dualizacija bozonske strune i tip II superstrune u prisustvu koordinatno zavisnih pozadinskih polja

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T-dualizacija bozonske strune i tip II superstrune u prisustvu koordinatno zavisnih pozadinskih polja

- Uvod
- Kratak kurs teorije struna
- Motivacija
- Rezultati
  - Bozonske strune
  - Superstrune

Uvod

[1] B. Nikolić, D. Obrić, *Noncommutativity and nonassociativity of type II superstring with coordinate dependent RR field — the general case*, JHEP 12 (2022) 078.

[2] B. Nikolić, D. Obrić, *Combined fermionic and bosonic T-duality of type II superstring theory with coordinate dependent RR field*, Fortschr. Phys. (2022) 2200160.

[3] B. Nikolić, D. Obrić, and B. Sazdović, *Noncommutativity and Nonassociativity of Type II Superstring with Coordinate Dependent RR Field*, Fortsch. Phys. 70 (2022) 2200048.

[4] B. Nikolić, D. Obrić, *Directly from H-flux to the family of three nonlocal R-flux theories*, JHEP 03 (2019) 136.

[5] B. Nikolić, D. Obrić, *Noncommutativity and nonassociativity of closed bosonic string on T-dual toroidal background*, Fortsch. Phys. 66 (2018) 040009.

Alternativa kvantnoj teoriji polja

Postoje dva načina da formulišemo relativističku kvantnu mehaniku

Kvantna mehanika
t parametar, $\boldsymbol{x}$ opertatori

Kvantna teorija polja
$\phi(t, \boldsymbol{x})$ operatori
$t, \boldsymbol{x}$ parametri

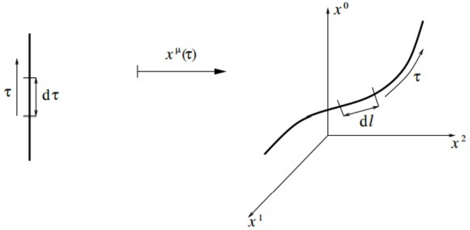
Kvantna mehanika
relativističke čestice
$t(\tau), \boldsymbol{x}(\tau)$ operatori
τ parametar

Relativistička čestica

Dejstvo koje opisuje trajektoriju bozonske čestice u zakrivljenom prostor-vremenu

$$S=-m\int ds, \quad ds=\sqrt{-G_{\mu\nu}(x)\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}d\tau.$$

$$S=\frac{1}{2}\int\left(\frac{1}{e}G_{\mu\nu}(x)\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}-em^2\right)d\tau.$$



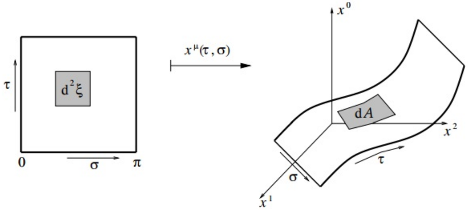
Trajektorija čestice

Bozonske strune

- Uopštenje relativističke čestice na objekat veće dimenzije, dodajemo još jedan parametar
- Dejstvo nastaje minimizacijom površine koju struna prebriše u prostor-vremenu

$$S=\kappa \int_\Sigma d\mu, \quad d\mu=\sqrt{-det(G_{\mu\nu}(x)\partial_m x^\mu \partial_n x^\nu)}d^2\xi=\sqrt{-det(G_{mn})}d^2\xi \quad (m,n=0,1).$$

$$S=\frac{\kappa}{2}\int_\Sigma d^2\xi\sqrt{-g}g^{mn}(\xi)G_{\mu\nu}(x)\partial_m x^\mu \partial_n x^\nu .$$



Trajektorija strune

Osobine teorije

Prednosti

- Jedan slobodan parametar, κ - zategnutost strune
- Opisuje gravitaciju
- Weyl-ova simetrija, $g^{mn}T_{mn}$

Mane

- 26 dimenziono prostor-vreme
- Nestabilan vakuum, tahioni
- Nedostaju fermionska stanja

Modifikacije pozadinskih polja

Pored gravitacionog polja $G_{\mu\nu}(x)$, možemo da dodamo i antisimetrično Kalb Ramondovo polje $B_{\mu\nu}(x)$ i dilatonsko polje $\Phi(x)$

$$S = \kappa \int_\Sigma d^2\xi \sqrt{-g} \left\{ \left[\frac{1}{2} g^{mn} G_{\mu\nu}(x) + \frac{\epsilon^{mn}}{\sqrt{-g}} B_{\mu\nu}(x) \right] \partial_m x^\mu \partial_n x^\nu + \Phi(x) R^{(2)} \right\}.$$

Wayl-ova simetrija zahteva da je

$$2\pi T^m_m = \beta^\Phi \sqrt{-g} R^{(2)} + \beta^G_{\mu\nu} \sqrt{-g} g^{mn} \partial_m x^\mu \partial_n x^\nu + \beta^B_{\mu\nu} \epsilon^{mn} \partial_m x^\mu \partial_n x^\nu,$$

$$\beta^G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4} B_{\mu\rho\sigma} B^{\rho\sigma}_\nu + 2 D_\mu a_\nu = 0,$$

$$\beta^B_{\mu\nu} \equiv D_\rho B^\rho_{\mu\nu} - 2 a_\rho B^\rho_{\mu\nu} = 0,$$

$$\beta^\Phi \equiv 2\pi\kappa \frac{D-26}{6} - R - \frac{1}{24} B_{\mu\rho\sigma} B^{\mu\rho\sigma} - D_\mu a^\mu + 4a^2 = c,$$

$$B_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}, \qquad a_\mu = \partial_\mu \Phi,$$

$$D^\nu \beta^G_{\nu\mu} + \partial_\mu \beta^\Phi = 0.$$

Superstrune

Relativistička čestica $\xrightarrow{+1 \text{ parametar}}$ Bozonska struna

$\downarrow$ Supersimetrija   Supersimetrija $\downarrow$

Supersimetrična čestica $\xrightarrow{+1 \text{ parametar}}$ Superstruna

U zavisnosti od tipa supersimetrije kao i od invarijantnosti na dodatne grupe simetrije postoji sledeća klasifikacija teorija struna

- tip I
- tip IIA i IIB
- heterotične $SO(32)$
- heterotične $E_8 \times E_8$

Osobine superstruna

Prednosti

- Jedan slobodan parametar, κ - zategnutost strune
- Opisuje sve čestice i interakcije
- Samo 5 koznistentnih teorija
- Nema tahiona

Mane

- 10 dimenziono prostor-vreme
- veliki broj konzistentnih vakuumskih stanja

Tip IIB superstrune u čistom spinorskom formalizmu

Jedna od mogućih formulacija supersimetrične teorije struna
Omogućava jednostavno uvođenje netrivijalnih pozadinskih polja
Dejstvo koje opisuje propagaciju strune u ravnom prostor-vremenu je

$$S=\int_\Sigma d^2\xi\Big(\tfrac{\kappa}{2}\eta_{\mu\nu}\partial_m x^\mu\partial_n x^\nu\eta^{mn}-\pi_\alpha\partial_-\theta^\alpha+\partial_+\bar\theta^\alpha\bar\pi_\alpha+\omega_\alpha\partial_-\lambda^\alpha+\bar\omega_\alpha\partial_+\bar\lambda^\alpha\Big),$$

$$\lambda^\alpha(\Gamma^\mu)_{\alpha\beta}\lambda^\beta=\bar\lambda(\Gamma^\mu)_{\alpha\beta}\bar\lambda^\beta=0\,.$$

Polja dodajemo preko integrisanog verteksnog operatora

$$V_{SG}=\int_\Sigma d^2\xi(X^T)^M A_{MN}\bar X^N.$$

$$X^M=\begin{pmatrix}\partial_+\theta^\alpha\\ \Pi_+^\mu\\ d_\alpha\\ \frac{1}{2}N_+^{\mu\nu}\end{pmatrix},\quad \bar X^M=\begin{pmatrix}\partial_-\bar\theta^\lambda\\ \Pi_-^\mu\\ \bar d_\lambda\\ \frac{1}{2}\bar N_-^{\mu\nu}\end{pmatrix},\quad A_{MN}=\begin{bmatrix}A_{\alpha\beta} & A_{\alpha\nu} & E_\alpha{}^\beta & \Omega_{\alpha,\mu\nu}\\ A_{\mu\beta} & A_{\mu\nu} & \bar E_\mu^\beta & \Omega_{\mu,\nu\rho}\\ E^\alpha{}_\beta & E_\nu^\alpha & P^{\alpha\beta} & C^\alpha{}_{\mu\nu}\\ \Omega_{\mu\nu,\beta} & \Omega_{\mu\nu,\rho} & \bar C^\beta{}_{\mu\nu} & S_{\mu\nu,\rho\sigma}\end{bmatrix}.$$

Tip IIB superstrune u čistom spinorskom formalizmu

$$d_\alpha=\pi_\alpha-\tfrac{1}{2}(\Gamma_\mu\theta)_\alpha\Big[\partial_+x^\mu+\tfrac{1}{4}(\theta\Gamma^\mu\partial_+\theta)\Big],\quad \bar{d}_\alpha=\bar{\pi}_\alpha-\tfrac{1}{2}(\Gamma_\mu\bar{\theta})_\alpha\Big[\partial_-x^\mu+\tfrac{1}{4}(\bar{\theta}\Gamma^\mu\partial_-\bar{\theta})\Big],$$

$$\Pi^\mu_+=\partial_+x^\mu+\tfrac{1}{2}\theta^\alpha(\Gamma^\mu)_{\alpha\beta}\partial_+\theta^\beta,\quad \Pi^\mu_-=\partial_-x^\mu+\tfrac{1}{2}\bar{\theta}^\alpha(\Gamma^\mu)_{\alpha\beta}\partial_-\bar{\theta}^\beta,$$

$$N^{\mu\nu}_+=\tfrac{1}{2}\omega_\alpha(\Gamma^{[\mu\nu]})^\alpha{}_\beta\lambda^\beta,\quad \bar{N}^{\mu\nu}_-=\tfrac{1}{2}\bar{\omega}_\alpha(\Gamma^{[\mu\nu]})^\alpha{}_\beta\bar{\lambda}^\beta.$$

Pozadinska polja zadovoljavaju sledeći skup jednačina

$$(1+\mathbf{D})A_{\alpha\beta}=(\Gamma^\mu\theta)_\alpha A_{\mu\beta},\quad (1+\bar{\mathbf{D}})A_{\alpha\beta}=(\Gamma^\mu\bar{\theta})_\beta A_{\alpha\mu},$$

$\uparrow$

16 redova

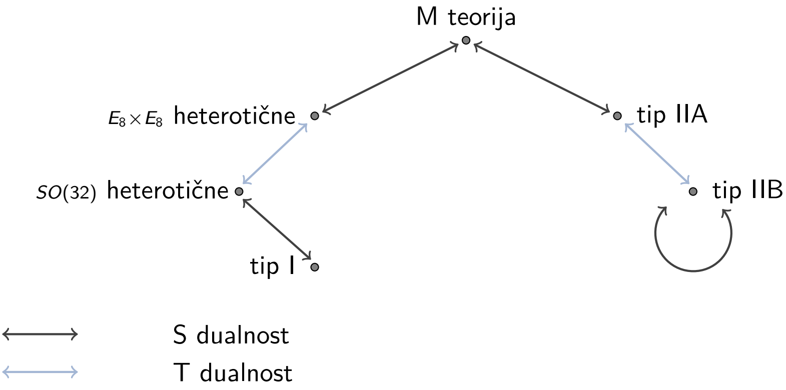
$\downarrow$

$$\mathbf{D}S_{\mu\nu,\mu_1\nu_1}=-(\Gamma_{[\mu}\theta)_\alpha\partial_{\nu]}C^\alpha{}_{\mu_1\nu_1},\quad \bar{\mathbf{D}}S_{\mu_1\nu_1\mu\nu,}=-(\Gamma_{[\mu}\bar{\theta})_\beta\partial_{\nu]}\bar{C}^\beta{}_{\mu_1\nu_1}.$$

$$\mathbf{D}\equiv\theta^\alpha\frac{\partial}{\partial\theta^\alpha},\quad \bar{\mathbf{D}}\equiv\bar{\theta}^\alpha\frac{\partial}{\partial\bar{\theta}^\alpha}$$

Mreža dualnosti - M teorija

Različiti tipovi superstruna nisu nezavisni, povezani su sa dve dualnosti



T-dualnost i Bušerova procedura

Suvišne prostorne dimenzije kompaktifikujemo

T-dualnost povezuje teorije koje imaju radius kompaktifikacije R sa teorijama gde je radius kompaktifikacije $1/R$

Bušerova procedura kao jedan od načina za dobijanje dualnih teorija

Postoje tri verzije procedure: standardna, uopštena i uopštenje uopštene

Bušerova procedura

Standardna procedura

- Potrebna izometrija
- Radi samo za koordinatno nezavisna pozadinska polja

Uopštena procedura

- Potrebna izometrija
- Radi za koordinatno zavisna pozadinska polja
- Nelokalna teorija

Uopštenje uopštene procedure

- Nije potrebna izometrija
- Radi za koordinatno zavisna pozadinska polja
- Nelokalna teorija

Bušerova procedura

Standardna procedura se sastoji od nekoliko koraka

- Pronalaženje izometrije: $S = \int_\Sigma d^2\xi \mathcal{L}(x, \partial x), \quad \delta x^\mu = \lambda^\mu \quad \to \quad \delta S = 0.$
- Lokalizacija izometrije:

$$\partial_m x^\mu \to D_m x^\mu = \partial_m x^\mu + v_m^\mu, \quad x^{inv} = \int_P d^m\xi D_m x^\mu, \quad S = \int_\Sigma d^2\xi \mathcal{L}(x^{inv}, Dx).$$

- Otklanjanje dodatnih stepeni slobode: $S_{add} = \int_\Sigma d^2\xi y_\mu \epsilon^{mn} \partial_m v_n^\mu.$
- Fiksiranje kalibracione simetrije:

$$S + S_{add} = \int_\Sigma d^2 \mathcal{L}(x, \partial x, y, v\partial v), \quad x(\xi) = constant \quad \to \quad S + S_{add} = \int_\Sigma d^2 \mathcal{L}(y, v, \partial v).$$

- Pronalaženjem jednačina kretanja za kalibraciona polja i ubacivanje rešenjem u dejstvo dobijam T-dualnu teoriju

Uopštena procedura: dodajemo invarijantne koordinate $\int_P d\xi^m D_m x^\mu$

Motivacija

Veće razumevanje T-dualnosti i fenomena koji potiču od nje

Izučavanje potrebnih uslova za dobijanje nekomutativnih teorija

Pokušaj da dobijemo bozonske koordinate preko komutacionih relacija fermionskih koordinata

Bozonske strune

T-dualizacija Bozonske strune

Radimo sa bozonskom strunom koja ima 3 kompaktifikovane koordinate

$$S=\kappa\int_{\Sigma}d^2\xi\sqrt{-g}\left\{\left[\frac{1}{2}g^{mn}G_{\mu\nu}(x)+\frac{\epsilon^{mn}}{\sqrt{-g}}B_{\mu\nu}(x)\right]\partial_m x^\mu\partial_n x^\nu+\Phi(x)R^{(2)}\right\},$$

Pozadinska polja opisuju geometriju torusa sa H fluksom

$$G_{\mu\nu}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\quad B_{\mu\nu}=\begin{pmatrix}0&Hz&0\\-Hz&0&0\\0&0&0\end{pmatrix}.$$

Uvodimo koordinate svetlosnog konusa $\xi^\pm=\frac{1}{2}(\tau\pm\sigma)$, $\partial_\pm=\partial_\tau\pm\partial_\sigma$.

$$\begin{aligned}S&=\kappa\int_{\Sigma}d^2\xi\,\partial_+x^\mu\Pi_{+\mu\nu}\partial_-x^\nu\\&=\kappa\int_{\Sigma}d^2\xi\Big[\frac{1}{2}(\partial_+x\partial_-x+\partial_+y\partial_-y+\partial_+z\partial_-z)+\partial_+xHz\partial_-y-\partial_+yHz\partial_-x\Big].\end{aligned}$$

T-dualizacija duž x ose - uvrnuti torus

Izometrija: $x \to x + a$

Primena Bušerove procedure $\partial_\pm x \to D_\pm x = \partial_\pm x + v_\pm, \quad \delta v_\pm = -\partial_\pm a,$

$$S_{add} = \frac{\kappa}{2} \int_\Sigma d^2\xi \gamma_1 (\partial_+ v_- - \partial_- v_+),$$

Fiksiranje simetrije $x = const$

$$S_{fix} = \kappa \int_\Sigma d^2\xi \left[\frac{1}{2}(v_+ v_- + \partial_+ y \partial_- y + \partial_+ z \partial_- z) + v_+ Hz \partial_- y - \partial_+ y Hz v_- + \frac{1}{2}\gamma_1 (\partial_+ v_- - \partial_- v_+) \right].$$

$$v_- = -\partial_- \gamma_1 - 2Hz\partial_- y, \ \ v_+ = \partial_+ \gamma_1 + 2Hz \partial_+ y \quad \to \quad {}_x S = \kappa \int_\Sigma d^2\xi \partial_+ ({}_x X)^\mu {}_x \Pi_{+\mu\nu} \partial_- ({}_x X)^\nu .$$

Pozadinska polja

$${}_x \Pi_{+\mu\nu} = {}_x B_{\mu\nu} + \frac{1}{2} {}_x G_{\mu\nu}, \quad {}_x B_{\mu\nu} = 0, \quad {}_x G_{\mu\nu} = \begin{pmatrix} 1 & 2Hz & 0 \\ 2Hz & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad {}_x X^\mu = \begin{pmatrix} \gamma_1 \\ y \\ z \end{pmatrix}.$$

T-dualizacija duž y ose - torus sa Q fluksom

Bušerova procedura daje

$$v_\pm = \pm\partial_\pm\gamma_2 - 2Hz\partial_\pm\gamma_1, \quad {}_{xy}S = \kappa\int_\Sigma d^2\xi\,\partial_+({}_{xy}X)^\mu\,{}_{xy}\Pi_{+\mu\nu}\partial_-({}_{xy}X)^\nu,$$

Pozadinska polja $\quad {}_{xy}\Pi_{+\mu\nu} = {}_{xy}B_{\mu\nu} + \frac{1}{2}\,{}_{xy}G_{\mu\nu}$

$$({}_{xy}X)^\mu = \begin{pmatrix}\gamma_1\\ \gamma_2\\ z\end{pmatrix}, \quad {}_{xy}B_{\mu\nu} = \begin{pmatrix}0 & -Hz & 0\\ Hz & 0 & 0\\ 0 & 0 & 0\end{pmatrix} = -B_{\mu\nu}, \quad {}_{xy}G_{\mu\nu} = \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}.$$

Da li imamo nekomutativnost?

$$\pi_x \cong \kappa\gamma_1', \quad \pi_y \cong \kappa\gamma_2' \quad \rightarrow \quad \{\gamma_1', \gamma_2'\} = 0 \quad \rightarrow \quad \{\gamma_1, \gamma_2\} = 0$$

Primena uopštene Bušerove procedure = standardna + invarijantne koordinate

$$z^{inv}=\int_P d\xi^m D_m z=\int_P d\xi^+ D_+ z+\int_P d\xi^- D_- z=z(\xi)-z(\xi_0)+\Delta V,$$
$$\Delta V=\int_P d\xi^m v_m=\int_P (d\xi^+ v_+ + d\xi^- v_-).$$

Odavde dobijamo sledeće jednačine kretanja i dualno dejstvo

$$v_\pm=\pm\partial_\pm\gamma_3-2\beta^\mp, \quad \beta^\pm=\pm\frac{1}{2}H(\gamma_1\partial_\mp\gamma_2-\gamma_2\partial_\mp\gamma_1),$$

$$_{xyz}S=\kappa\int_\Sigma d^2\xi\partial_+(_{xyz}X)^\mu{}_{xyz}\Pi_{+\mu\nu}\partial_-(_{xyz}X)^\nu.$$

Pozadinska polja

$$_{xyz}x^\mu=\begin{pmatrix}\gamma_1\\ \gamma_2\\ \gamma_3\end{pmatrix}, \quad {}_{xyz}B_{\mu\nu}=\begin{pmatrix}0 & -H\Delta V & 0\\ H\Delta V & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \quad {}_{xyz}G_{\mu\nu}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}.$$

Nekomutativne relacije

Koristimo zakone transformacija prepisane u kanonski oblik

$$\gamma_1' \cong \frac{1}{\kappa}\pi_x, \quad \gamma_2' \cong \frac{1}{\kappa}\pi_y \quad \gamma_3' \cong \frac{1}{\kappa}\pi_z - H(xy' - yx'),$$

kao i Poasonove zagrade početne teorije

$$\{x^\mu(\sigma), \pi_\nu(\bar\sigma)\} = \delta^\mu_\nu \delta(\sigma - \bar\sigma), \quad \{x^\mu, x^\nu\} = \{\pi_\mu, \pi_\nu\} = 0.$$

Odavde sledi

$$\{\gamma_1(\sigma), \gamma_3(\bar\sigma)\} \cong -\frac{H}{\kappa}[2y(\sigma) - y(\bar\sigma)]\bar H(\sigma - \bar\sigma), \quad \{\gamma_2(\sigma), \gamma_3(\bar\sigma)\} \cong \frac{H}{\kappa}[2x(\sigma) - x(\bar\sigma)]\bar H(\sigma - \bar\sigma),$$

$$\{\gamma_1(\sigma+2\pi), \gamma_3(\sigma)\} \cong -\frac{H}{\kappa}[4\pi N_y + y(\sigma)], \quad \{\gamma_2(\sigma+2\pi), \gamma_3(\sigma)\} \cong \frac{H}{\kappa}[4\pi N_x + x(\sigma)],$$

Uvodimo brojeve namotaja $x(\sigma+2\pi) - x(\sigma) = 2\pi N_x, \quad y(\sigma+2\pi) - y(\sigma) = 2\pi N_y.$

Neasocijativnost

Možemo da nađemo i neasocijativne relacije kao

$$\{\gamma_1(\sigma_1),\gamma_2(\sigma_2),\gamma_3(\sigma_3)\}\equiv$$

$$\{\gamma_1(\sigma_1),\{\gamma_2(\sigma_2),\gamma_3(\sigma_3)\}\}+\{\gamma_2(\sigma_2),\{\gamma_3(\sigma_3),\gamma_1(\sigma_1)\}\}+\{\gamma_3(\sigma_3),\{\gamma_1(\sigma_1),\gamma_2(\sigma_2)\}\}$$

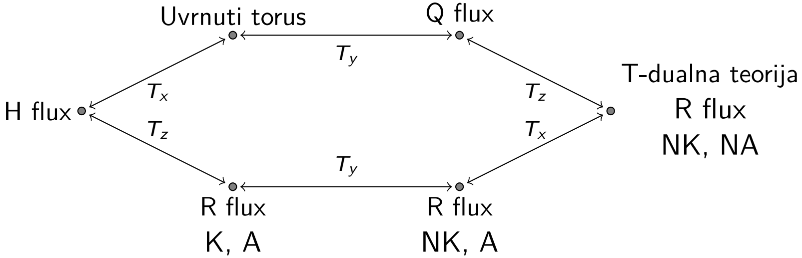
$$\cong-\frac{2H}{\kappa^2}\Big[\bar{H}(\sigma_1-\sigma_2)\bar{H}(\sigma_2-\sigma_3)+\bar{H}(\sigma_2-\sigma_1)\bar{H}(\sigma_1-\sigma_3)+\bar{H}(\sigma_1-\sigma_3)\bar{H}(\sigma_3-\sigma_2)\Big].$$

U slučaju kad je $\sigma_2=\sigma_3=\sigma$ i $\sigma_1=\sigma+2\pi$ imamo

$$\{\gamma_1(\sigma+2\pi),\gamma_2(\sigma),\gamma_3(\sigma)\}\cong\frac{2H}{\kappa^2}.$$

Mreža teorija

Modifikacija redosleda T-dualizacije proizvodi različite teorije



T_i - T-dualizacija duž i pravca
(N)K - (Ne)Komutativna teorija
(N)A - (Ne)Asocijativna teorija

Poasonove zagrade i zakoni transformacija su isti

Rezultati

Superstrune

T-dualizacija tip IIB superstrune - bozonske koordinate

Dualizujemo duž svih bozonkih koordinata odjednom

$$S = S_0 + V_{SG}, \quad V_{SG} = \int_\Sigma d^2\xi (X^T)^M A_{MN} \bar{X}^N.$$

$$S_0 = \int_\Sigma d^2\xi \Big(\frac{\kappa}{2} \eta_{\mu\nu} \partial_m x^\mu \partial_n x^\nu \eta^{mn} - \pi_\alpha \partial_- \theta^\alpha + \partial_+ \bar{\theta}^\alpha \bar{\pi}_\alpha \Big) + S_\lambda + S_{\bar{\lambda}} \,,$$

Biramo sledeća pozadinska polja

$$A_{MN} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa(\frac{1}{2} g_{\mu\nu} + B_{\mu\nu}) & \bar{\Psi}_\mu^\beta & 0 \\ 0 & -\Psi_\nu^\alpha & \frac{2}{\kappa}(f^{\alpha\beta} + C_\rho^{\alpha\beta} x^\rho) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; \Pi_\pm^\mu \to \partial_\pm x^\mu, \quad d_\alpha \to \pi_\alpha, \quad \bar{d}_\alpha \to \bar{\pi}_\alpha.$$

T-dualizacija tip IIB superstrune - bozonske koordinate

Implementacijom datih polja imamo sledeće dejstvo

$$S=\kappa \int_\Sigma d^2\xi \Big[\Pi_{+\mu\nu}\partial_+ x^\mu \partial_- x^\nu + \frac{1}{2}(\partial_+\bar\theta^\alpha + \partial_+ x^\mu \bar\Psi^\alpha_\mu)\big(F^{-1}(x)\big)_{\alpha\beta}(\partial_-\theta^\beta + \Psi^\beta_\nu \partial_- x^\nu)\Big].$$

$$F^{\alpha\beta}(x)=f^{\alpha\beta}+C^{\alpha\beta}_\mu x^\mu\,, \quad (F^{-1}(x))_{\alpha\beta}=(f^{-1})_{\alpha\beta}-(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_\mu (f^{-1})_{\beta_1\beta}\, x^\mu\,.$$

Bušerova procedura daje

$$^bS=\frac{\kappa}{2}\int_\Sigma d^2\xi\Big[\frac{1}{2}\bar\Theta^{\mu\nu}\partial_+ y_\mu \partial_- y_\nu + \partial_+\bar\theta^\alpha (^bF^{-1}\big(V^{(0)}\big))_{\alpha\beta}\partial_-\theta^\beta$$

$$+\partial_+ y_\mu\, {}^b\bar\Psi^{\mu\alpha}(^bF^{-1}\big(V^{(0)}\big))_{\alpha\beta}\partial_-\theta^\beta + \partial_+\bar\theta^\alpha(^bF^{-1}\big(V^{(0)}\big))_{\alpha\beta}\, {}^b\Psi^{\nu\beta}\partial_- y_\nu\Big],$$

Zakoni transformacija

$$\partial_+ y_\mu \cong 2\Big[\partial_+ x^\nu \bar\Pi_{+\nu\mu} + \beta^-_\mu(x)\Big] + \partial_+\bar\theta^\alpha (F^{-1}(x))_{\alpha\beta}\Psi^\beta_\mu$$

$$\partial_- y_\mu \cong -2\Big[\bar\Pi_{+\mu\nu}\partial_- x^\nu + \beta^+_\mu(x)\Big] - \bar\Psi^\alpha_\mu (F^{-1}(x))_{\alpha\beta}\partial_-\theta^\beta$$

$$\Delta V^{(0)\rho}=\int_P d\xi^m v^{(0)\rho}_m.$$

T-dualizacija tip IIB superstrune - bozonske koordinate

$\beta^\pm$ funkcije

$$\beta^-_\mu(x) = \frac{1}{4}\partial_+\left[\bar\theta^\alpha + x^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\left[\theta^\beta + \Psi^\beta_{\nu_2}x^{\nu_2}\right]$$
$$-\frac{1}{4}\left[\bar\theta^\alpha + x^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\partial_+\left[\theta^\beta + \Psi^\beta_{\nu_2}x^{\nu_2}\right],$$
$$\beta^+_\mu(x) = \frac{1}{4}\left[\bar\theta^\alpha + x^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\partial_-\left[\theta^\beta + \Psi^\beta_{\nu_2}x^{\nu_2}\right]$$
$$-\frac{1}{4}\partial_-\left[\bar\theta^\alpha + x^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\left[\theta^\beta + \Psi^\beta_{\nu_2}x^{\nu_2}\right].$$

Dualna pozadinska polja

$$\bar\Pi_{+\mu\nu} = \Pi_{+\mu\nu} + \frac{1}{2}\bar\Psi^\alpha_\mu(F^{-1}(x))_{\alpha\beta}\Psi^\beta_\nu, \quad \check\Pi_{+\mu\nu} = \Pi_{+\mu\nu} + \frac{1}{2}\bar\Psi^\alpha_\mu(f^{-1})_{\alpha\beta}\Psi^\beta_\nu,$$
$$\bar\Theta^{\mu\nu}_- = \check\Theta^{\mu\nu}_- + \frac{1}{2}\check\Theta^{\mu\mu_1}_-\bar\Psi^\alpha_{\mu_1}(f^{-1})_{\alpha\alpha_1} C_\rho^{\alpha_1\beta_1} V^{(0)\rho}(f^{-1})_{\beta_1\beta}\Psi^{\beta_1}_{\nu_1}\check\Theta^{\nu_1\nu}_-,$$
$$\check\Theta^{\mu\nu}_-\check\Pi_{\nu\rho} = \delta^\mu_\rho, \qquad \check\Theta^{\mu\nu}_- = \Theta^{\mu\nu}_- - \frac{1}{2}\Theta^{\mu\mu_1}_-\bar\Psi^\alpha_{\mu_1}(\bar f^{-1})_{\alpha\beta}\Psi^\beta_{\nu_1}\Theta^{\nu_1\nu}_-$$
$$\bar f^{\alpha\beta} = f^{\alpha\beta} + \frac{1}{2}\Psi^\alpha_\mu\Theta^{\mu\nu}_-\bar\Psi^\beta_\nu, \quad \Theta^{\mu\nu}_-\Pi_{+\mu\rho} = \delta^\mu_\rho, \qquad \Theta_- = -4(G_E^{-1}\Pi_- G^{-1})^{\mu\nu}.$$

$${}^b\bar\Pi^{\mu\nu}_+ = \frac{1}{4}\bar\Theta^{\mu\nu}_-, \quad {}^b\bar\Psi^{\mu\alpha} = \frac{1}{2}\Theta^{\mu\nu}_-\bar\Psi^\alpha_\mu, \quad {}^b\Psi^{\nu\beta} = -\frac{1}{2}\Psi^\beta_\mu\Theta^{\mu\nu}_-,$$
$${}^bF^{\alpha\beta}(V^{(0)}) = F^{\alpha\beta}(V^{(0)}) + \frac{1}{2}\Psi^\alpha_\mu\Theta^{\mu\nu}_-\bar\Psi^\beta_\nu.$$

Bozonske nekomutativne relacije

Kanonski zakoni transformacija

$$y'_\mu \cong \frac{\pi_\mu}{\kappa} + \beta^0_\mu(x),$$

$$\beta^0_\mu(x) = \frac{1}{2}\partial_\sigma \left[\bar\theta^\alpha + x^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\left[\theta^\beta + \Psi^\beta_{\nu_2}x^{\nu_2}\right]$$
$$-\frac{1}{2}\left[\bar\theta^\alpha + x^{\nu_1}\bar\Psi^\alpha_{\nu_1}\right](f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\partial_\sigma\left[\theta^\beta + \Psi^\beta_{\nu_2}x^{\nu_2}\right].$$

Nekomutativne relacije

$$\{y_{\nu_1}(\sigma), y_{\nu_2}(\bar\sigma)\} \cong \frac{1}{2\kappa}[2\delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2} - \delta^{\mu_2}_{\nu_1}\delta^{\mu_1}_{\nu_2}]\left[K_{\mu_1\mu_2}(\bar\sigma) + K_{\mu_2\mu_1}(\sigma)\right]\bar H(\sigma - \bar\sigma),$$

$$K_{\mu\nu}(\sigma) = (\bar\theta^\alpha(\sigma) + x^{\mu_1}(\sigma)\bar\Psi^\alpha_{\mu_1})(f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\Psi^\beta_\nu$$
$$-\bar\Psi^\alpha_\nu(f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}(\theta^\beta(\sigma) + \Psi^\beta_{\nu_1}x^{\nu_1}(\sigma)).$$

$$\{\theta^\alpha(\sigma), y_\mu(\bar\sigma)\} \cong 0, \qquad \{\bar\theta^\alpha(\sigma), y_\mu(\bar\sigma)\} \cong 0.$$

Bozonske neasocijativne relacije

Za neasocijativnost dobijamo

$$\{y_\nu(\sigma),\{y_{\nu_1}(\sigma_1),y_{\nu_2}(\sigma_2)\}\}\cong\frac{1}{2\kappa}\bar{H}(\sigma_1-\sigma_2)\bar{\Psi}^\alpha_{\mu_1}(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_{\mu_2}(f^{-1})_{\beta_1\beta}\Psi^\beta_{\mu_3}$$
$$\times\Big[\bar{H}(\sigma_1-\sigma)[2\delta^{\mu_1}_\nu\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu_1}-2\delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_\nu-\delta^{\mu_1}_\nu\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_{\nu_2}+\delta^{\mu_1}_{\nu_2}\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_\nu]$$
$$+\bar{H}(\sigma_2-\sigma)[2\delta^{\mu_1}_\nu\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_{\nu_2}-2\delta^{\mu_1}_{\nu_2}\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_\nu-\delta^{\mu_1}_\nu\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu_1}+\delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_\nu]\Big].$$

Stavljanjem $\sigma=\sigma_2=\bar{\sigma}$ i $\sigma_1=\bar{\sigma}+2\pi$ dobijamo

$$\{y_\nu(\bar{\sigma}),\{y_{\nu_1}(\bar{\sigma}+2\pi),y_{\nu_2}(\bar{\sigma})\}\}\cong$$
$$\bar{\Psi}^\alpha_{\mu_1}(f^{-1})_{\alpha\alpha_1}C^{\alpha_1\beta_1}_{\mu_2}(f^{-1})_{\beta_1\beta}\Psi^\beta_{\mu_3}[2\delta^{\mu_1}_\nu\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_{\nu_1}-2\delta^{\mu_1}_{\nu_1}\delta^{\mu_2}_{\nu_2}\delta^{\mu_3}_\nu-\delta^{\mu_1}_\nu\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_{\nu_2}+\delta^{\mu_1}_{\nu_2}\delta^{\mu_2}_{\nu_1}\delta^{\mu_3}_\nu].$$

T-dualizacija tip IIB superstrune - fermionske koordinate

U dejstvu nemamo članove koji sadrže $\partial_+\theta^\alpha$ i $\partial_-\bar{\theta}^\alpha$. Ovo znači da imamo dodatnu simetriju koju moramo da fiksiramo

Koristimo BRST formalizam

$$\delta\theta^\alpha = \epsilon^\alpha(\sigma^+), \quad \delta\bar{\theta}^\alpha = \bar{\epsilon}^\alpha(\sigma^-), \quad (\sigma^\pm = \tau \pm \sigma).$$

$$s\theta^\alpha = c^\alpha(\sigma^+), \qquad s\bar{\theta}^\alpha = \bar{c}^\alpha(\sigma^-), \quad sC_\alpha = b_{+\alpha}, \quad s\bar{C}_\alpha = \bar{b}_{-\alpha}, \quad sb_{+\alpha} = 0, \quad s\bar{b}_{-\alpha} = 0.$$

Uvodimo kalibracioni fermion

$$\Psi = \frac{\kappa}{2}\int_\Sigma d^2\xi \left[\bar{C}_\alpha\left(\partial_+\theta^\alpha + \frac{1}{2}\alpha^{\alpha\beta}b_{+\beta}\right) + \left(\partial_-\bar{\theta}^\alpha + \frac{1}{2}\bar{b}_{-\beta}\alpha^{\beta\alpha}\right)C_\alpha\right],$$

Odavde dobijamo

$$S_{gf} = -\frac{\kappa}{2}\int_\Sigma d^2\xi\, \partial_-\bar{\theta}^\alpha(\alpha^{-1})_{\alpha\beta}\partial_+\theta^\beta, \quad S_{F-P} = \frac{\kappa}{2}\int_\Sigma d^2\xi \left[\bar{C}_\alpha\partial_+c^\alpha + (\partial_-\bar{c}^\alpha)C_\alpha\right].$$

Nakon dodavanja člana koji fiksira kalibracionu simetriju Bušerova procedura je ista

$$ {}^{bf}S=\kappa\int_\Sigma d^2\xi\Big[\frac{1}{4}\Theta_-^{\mu\nu}\partial_+y_\mu\partial_-y_\nu-\frac{1}{4}\partial_+y_\mu\Theta_-^{\mu\nu}\bar{\Psi}_\nu^\alpha\partial_-z_\alpha-\frac{1}{4}\partial_+\bar{z}_\alpha\Psi_\mu^\alpha\Theta_-^{\mu\nu}\partial_-y_\nu $$

$$ +\frac{1}{2}\partial_+\bar{z}_\alpha{}^bF^{\alpha\beta}(V^{(0)})\partial_-z_\beta-\frac{1}{2}\partial_-\bar{z}_\alpha(\alpha)^{\alpha\beta}\partial_+z_\beta\Big]. $$

Zakoni transformacija

$$ \partial_+\theta^\alpha=-(\alpha)^{\alpha\beta}\partial_+z_\beta,\qquad \partial_-\bar{\theta}^\beta=\partial_-\bar{z}_\alpha(\alpha)^{\alpha\beta}, $$

$$ \partial_+\bar{\theta}^\alpha=-\bar{Z}_{+\beta}\ {}^bF^{\beta\alpha}(V^{(0)})-\beta_\nu^-(V^{(0)},U^{(0)})\ {}^b\bar{\Psi}^{\nu\alpha}, $$

$$ \partial_-\theta^\beta=-{}^bF^{\beta\alpha}(V^{(0)})Z_{-\alpha}-\beta_\mu^+(V^{(0)},U^{(0)})\ {}^b\Psi^{\mu\beta} $$

$$ ({}^bF^{-1}(V^{(0)}))_{\alpha\beta}{}^b\Psi^{\nu\beta}\partial_-y_\nu+\partial_-z_\alpha=Z_{-\alpha},\qquad \partial_+y_\mu{}^b\bar{\Psi}^{\mu\alpha}({}^bF^{-1}(V^{(0)}))_{\alpha\beta}-\partial_+\bar{z}_\beta=\bar{Z}_{+\beta}. $$

$$ \beta_\mu^\pm(x,\theta)=\mp\frac{1}{8}\partial_\mp\left[\bar{\theta}^\alpha+x^{\nu_1}\bar{\Psi}_{\nu_1}^\alpha\right](f^{-1})_{\alpha\alpha_1}C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\left[\theta^\beta+\Psi_{\nu_2}^\beta x^{\nu_2}\right] $$

$$ \pm\frac{1}{8}\left[\bar{\theta}^\alpha+x^{\nu_1}\bar{\Psi}_{\nu_1}^\alpha\right](f^{-1})_{\alpha\alpha_1}C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\partial_\mp\left[\theta^\beta+\Psi_{\nu_2}^\beta x^{\nu_2}\right]. $$

Fermionske nekomutativne relacije

Fermionski zakoni transformacije u kanonskom obliku

$$\partial_\sigma z_\alpha \cong \frac{1}{\kappa}\bar{\pi}_\alpha, \quad \partial_\sigma \bar{z}_\alpha \cong -\frac{1}{\kappa}\pi_\alpha.$$

Koristimo sledeće Poasonove zagrade

$$\{x^\mu(\sigma), \pi_\nu(\bar{\sigma})\} = \delta^\mu_\nu \delta(\sigma - \bar{\sigma}), \quad \{\theta^\alpha(\sigma), \pi_\beta(\bar{\sigma})\} = \{\bar{\theta}^\alpha(\sigma), \bar{\pi}_\beta(\bar{\sigma})\} = -\delta^\alpha_\beta \delta(\sigma - \bar{\sigma}),$$

Nakon fermionske T-dualizacije dobijamo sledeće nekomutacione relacije

$$\{y_\mu(\sigma), \bar{z}_\beta(\bar{\sigma})\} \cong$$

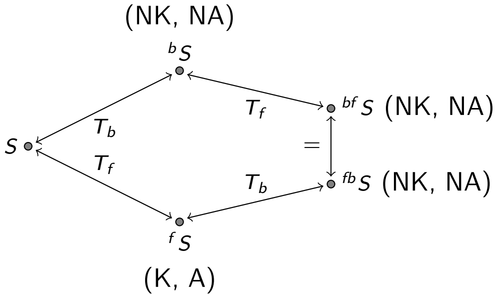
$$\frac{1}{2\kappa}\left[\bar{\theta}^\alpha(\sigma) + x^{\nu_1}(\sigma)\bar{\Psi}^\alpha_{\nu_1} - 2\left(\bar{\theta}^\alpha(\bar{\sigma}) + x^{\nu_1}(\bar{\sigma})\bar{\Psi}^\alpha_{\nu_1}\right)\right](f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\bar{H}(\sigma - \bar{\sigma}),$$

$$\{y_\mu(\sigma), z_\alpha(\bar{\sigma})\} \cong$$

$$\frac{1}{2\kappa}(f^{-1})_{\alpha\alpha_1} C_\mu^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\left[\theta^\beta(\sigma) + \Psi^\beta_{\nu_2}x^{\nu_2}(\sigma) - 2\left(\theta^\beta(\bar{\sigma}) + \Psi^\beta_{\nu_2}x^{\nu_2}(\bar{\sigma})\right)\right]\bar{H}(\sigma - \bar{\sigma}).$$

Promena redosleda T-dualizacije

T-dualizacija je mogla da se radi i u drugom smeru



T_b - Bozonska dualizacija
T_f - Fermionska dualizacija
(N)K - (Ne)Komutativna teorija
(N)A - (Ne)Asocijativna teorija

Bozonska T-dualizacija opšteg slučaja RR polja

Korišćenjem RR polja koje ima i simetričan deo dosta komplikuje račun

Teorija više nije invarijantna na translacije

$\beta^{\pm}$ funkcije postaju nelokalne i zavise od puta koji uvodimo kad dodajemo invarijantne koordinate

Komutacione relacije se modifikuju

T-dualna teorija je i dalje ista

Nekomutativne relacije

Nove Poasonove zagrade

$$\{y_{\nu_1}(\sigma),y_{\nu_2}(\bar{\sigma})\}\cong\frac{2}{k}(\check{\Pi}_{+}+\check{\Pi}_{+}^{T})^{-1\mu_1\mu_2}\left[G_{\nu_1\mu_1}B_{\nu_2\mu_2}+B_{\nu_1\mu_1}G_{\nu_2\mu_2}\right]H(\sigma-\bar{\sigma})$$
$$+\frac{1}{k}\bar{\Psi}^{\alpha}_{\nu_3}(f^{-1})_{\alpha\alpha_1}G_{\rho}^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\Psi^{\beta}_{\nu_4}(\delta^{\nu_3}_{\mu_3}\delta^{\nu_4}_{\mu_4}+\delta^{\nu_4}_{\mu_3}\delta^{\nu_3}_{\mu_4})(\check{\Pi}_{+}+\check{\Pi}_{+}^{T})^{-1\mu_3\mu_1}(\check{\Pi}_{+}+\check{\Pi}_{+}^{T})^{-1\mu_4\mu_2}$$
$$\times\left[G_{\nu_1\mu_1}B_{\nu_2\mu_2}x^{\rho}(\bar{\sigma})+B_{\nu_1\mu_1}G_{\nu_2\mu_2}x^{\rho}(\sigma)\right]\bar{H}(\sigma-\bar{\sigma}).$$

Nova neasocijativna relacija

$$\{y_{\nu}(\sigma),\{y_{\nu_1}(\sigma_1),y_{\nu_2}(\sigma_2)\}\}\cong\frac{G_{\nu\mu}}{k^2}(\check{\Pi}_{+}+\check{\Pi}_{+}^{T})^{-1\rho\mu}$$
$$\times\bar{\Psi}^{\alpha}_{\nu_3}(f^{-1})_{\alpha\alpha_1}C_{\rho}^{\alpha_1\beta_1}(f^{-1})_{\beta_1\beta}\Psi^{\beta}_{\nu_4}(\delta^{\nu_3}_{\mu_3}\delta^{\nu_4}_{\mu_4}+\delta^{\nu_4}_{\mu_3}\delta^{\nu_3}_{\mu_4})(\check{\Pi}_{+}+\check{\Pi}_{+}^{T})^{-1\mu_3\mu_1}(\check{\Pi}_{+}+\check{\Pi}_{+}^{T})^{-1\mu_4\mu_2}$$
$$\times\left[G_{\nu_1\mu_1}B_{\nu_2\mu_2}\bar{H}(\sigma-\sigma_2)+B_{\nu_1\mu_1}G_{\nu_2\mu_2}\bar{H}(\sigma-\sigma_1)\right]\bar{H}(\sigma_1-\sigma_2).$$

Stavljanjem $\sigma_1=\sigma_2=\bar{\sigma}$ i $\sigma=\bar{\sigma}+2\pi$

$$\{y_{\nu}(\bar{\sigma}+2\pi),\{y_{\nu_1}(\bar{\sigma}),y_{\nu_2}(\bar{\sigma})\}\}\cong 0.$$

Zaključak

Da bi dobili nekomutativnu teoriju zatvorenih struna potrebno je da imamo koordinatno zavisna polja

Nekomutativnost se javlja samo kad izvršimo dualizaciju duž koordinata od kojih pozadinska polja zavise

Za dobijanje fermionskih nekomutativnih relacija potreba su pozadinska polja koja zavise od fermionskih koordinata

Potpuno dualizovana teorija ne zavisi od redosleda T-dualizacije

Rad sa simetričnim poljima menja nekomutacione relacije dok dualna teorija ostaje ista

Kraj

Hvala na pažnji